New test - January 16, 2018

1a. *[4 marks]*

Let $f(x) = \sqrt{4x+5}$, for $x \geqslant -1.25$. $(4x+5)^{1/2}$

Find $f'(1)$. $= \boxed{\frac{1}{2}(4x+5)^{-\frac{1}{2}}}$

1b. *[2 marks]*

Consider another function g. Let R be a point on the graph of g. The x-coordinate of R is 1. The equation of the tangent to the graph at R is $y = 3x + 6$. $g'(x) = 3x+6$

$g'(1) = 3+6 = 9$

Write down $g'(1)$. $= \boxed{9}$

~~**1c.**~~ *[2 marks]*

Find $g(1)$. $\frac{3}{2}x^2 + 6x$

~~**1d.**~~ *[7 marks]*

Let $h(x) = f(x) \times g(x)$. Find the equation of the tangent to the graph of h at the point where $x = 1$.

2a. *[3 marks]*

Let $f'(x) = -24x^3 + 9x^2 + 3x + 1$. $f''(x) = -72x^2 + 18x + 3 = 3(-24x^2 + 6x + 1)$

$x = \frac{-6 \pm \sqrt{6^2 - 4(-24)(1)}}{2(-24)} = \frac{-6 + \sqrt{36+96}}{-48} = \frac{-6 \pm \sqrt{132}}{-48}$

There are two points of inflexion on the graph of f. Write down the x-coordinates of these points. $= -.114$ or $= 0.364$

2b. *[2 marks]*

Let $g(x) = f''(x)$. Explain why the graph of g has no points of inflexion.

$g(x)$ is a quadratic - $g'(x)$ is linear and $g''(x)$ is a constant

no change in sign $\Rightarrow$ no inflexion points

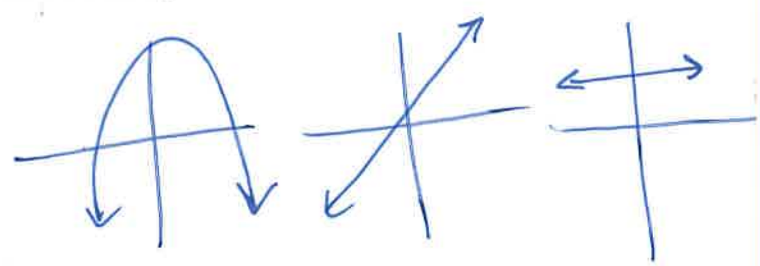
3a. *[8 marks]*

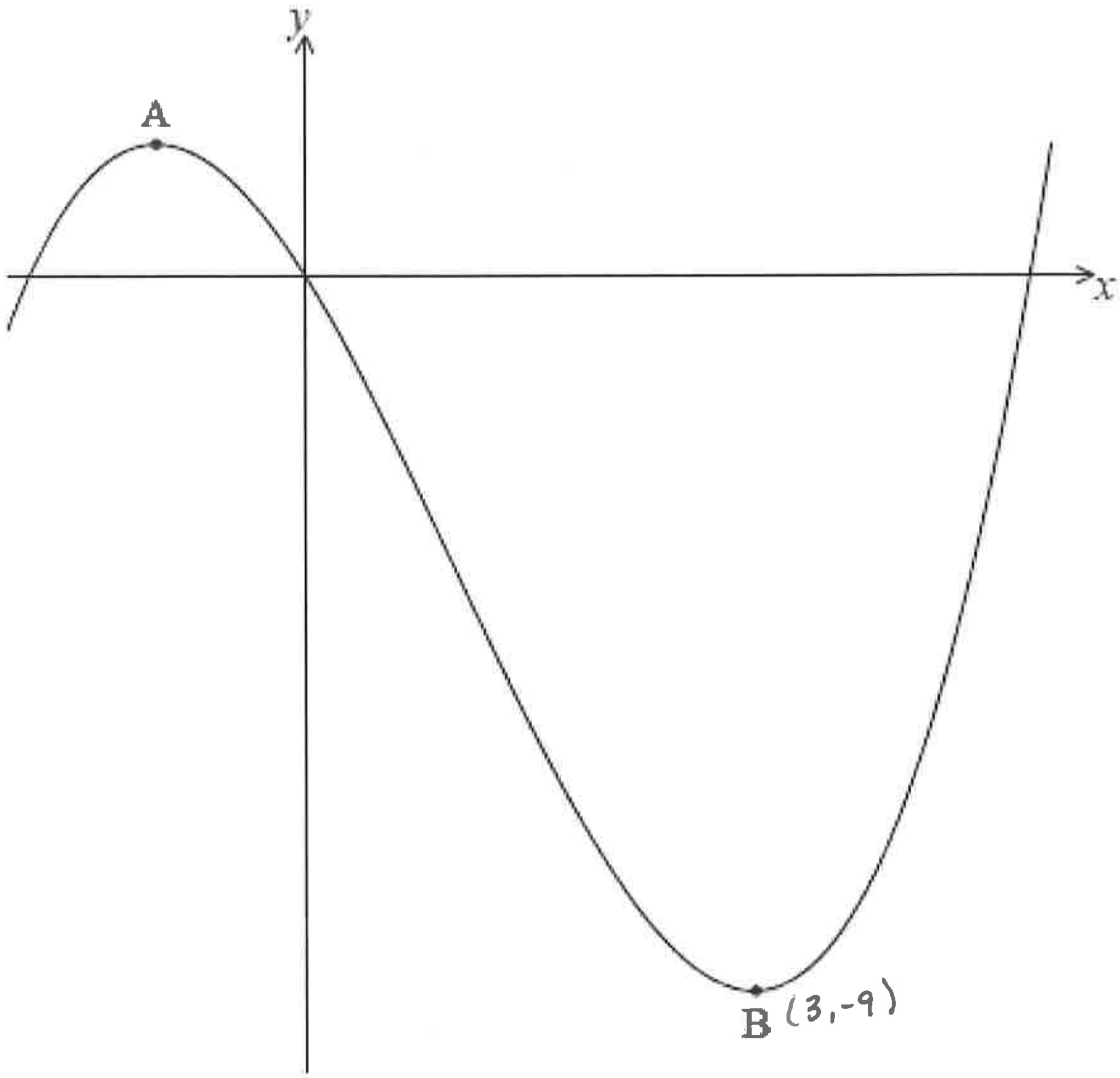
Let $f(x) = \cancel{\frac{1}{2}}\frac{1}{3}x^3 - x^2 - 3x$. Part of the graph of f is shown below.

$f'(x) = x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3,\ x = -1$





There is a maximum point at A and a minimum point at B(3, – 9) .

Find the coordinates of A. $f(-1) = \frac{1}{3}(-1)^3 - 2(-1)^2 - 3(-1) = -\frac{1}{3} + 3 = -\frac{2}{3}$

$-2 \quad +3$

3b. *[6 marks]*

Write down the coordinates of

(i) the image of B after reflection in the *y*-axis; $(-3, -9)$

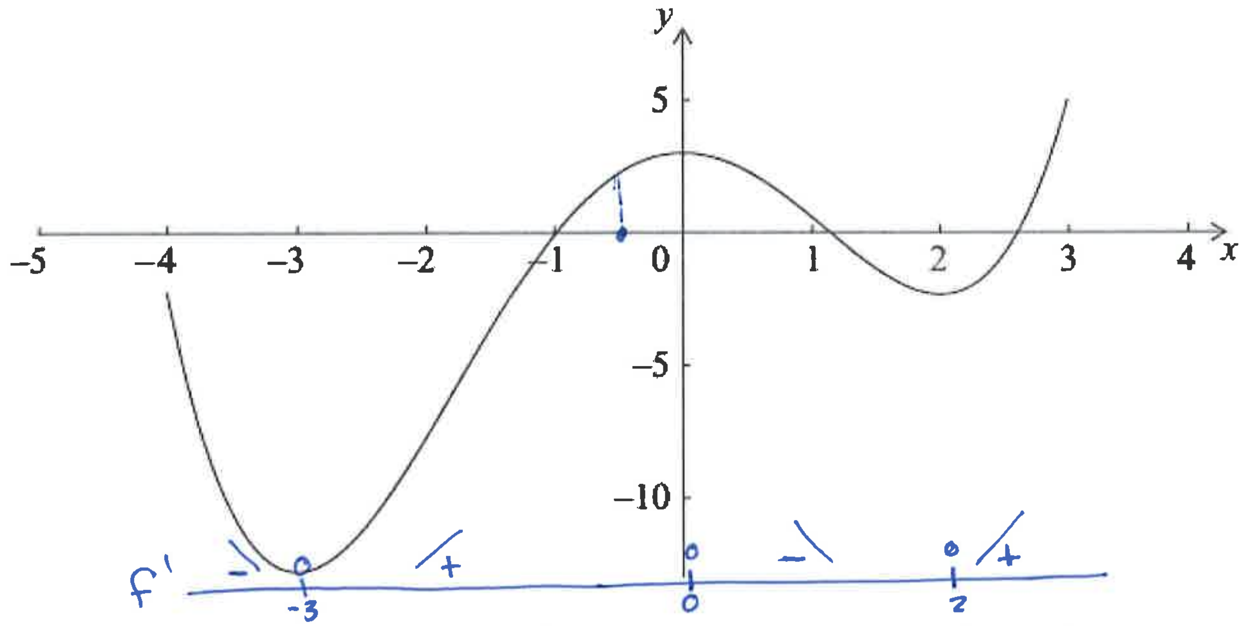
(ii) the image of B after translation by the vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$; $(1, -4)$

(iii) the image of B after reflection in the *x*-axis followed by a horizontal stretch with scale factor $\frac{1}{2}$.

$(3, 9) \rightarrow (\frac{3}{2}, 9)$

4a. *[2 marks]*

A function *f* is defined for $-4 \leq x \leq 3$. The graph of *f* is given below.



The graph has a local maximum when $x = 0$, and local minima when $x = -3$, $x = 2$.

f'' +

Write down the *x*-intercepts of the graph of the **derivative** function, f'. $-3, 0, 2$

4b. *[2 marks]*

Write down all values of *x* for which $f'(x)$ is positive. $(-3, 0)$ and $(2, 3)$

4c. *[2 marks]*

At point D on the graph of *f*, the *x*-coordinate is -0.5. Explain why $f''(x) < 0$ at D.

f is concave down at $x = -0.5 \Rightarrow f''(x) < 0$

5a. *[3 marks]*

Consider the function *f* with second derivative $f''(x) = 3x - 1$. The graph of *f* has a minimum point at A(2, 4) and a maximum point at $\mathrm{B}\left(-\frac{4}{3}, \frac{358}{27}\right)$.

Use the second derivative to justify that B is a maximum.

$f''\left(-\frac{4}{3}\right) = 3\left(-\frac{4}{3}\right) - 1 = -4 - 1 = -5 < 0$

$\therefore f''\left(-\frac{4}{3}\right) < 0 \Rightarrow f$ is concave up $\Rightarrow$ B is a maximum

5b. *[4 marks]*

Given that $f'(x) = \frac{3}{2}x^2 - x + p$, show that $p = -4$.

$f(x) = \frac{1}{3}\cdot\frac{3}{2}x^3 - \frac{1}{2}x^2 + px$

$= \frac{1}{2}x^3 - \frac{1}{2}x^2 - 4x + C$

$\frac{3}{2}x^2 - x + p = 0$

$\frac{3}{2}(2)^2 - (2) = -p$

$6 - 2 = -p$

$4 = -p \Rightarrow p = -4$

$f'(x) = \frac{3}{2}x - x + p$ show $p = -4$

~~5c~~. *[7 marks]*

Find $f(x)$.

6a. *[2 marks]*

Consider $f(x) = x^2 + \frac{p}{x}$, $x \neq 0$, where p is a constant.

$f(x) = x^2 + px^{-1}$

$f'(x) = 2x - px^{-2}$

Find $f'(x)$.

6b. *[4 marks]*

$2(-2) - p(-2)^{-2} = 0$

$-4 - \frac{p}{4} = 0$

$-\frac{p}{4} = 4$

$-p = 16$

$p = -16$

There is a minimum value of $f(x)$ when $x = -2$. Find the value of p.

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