Key (handwritten)
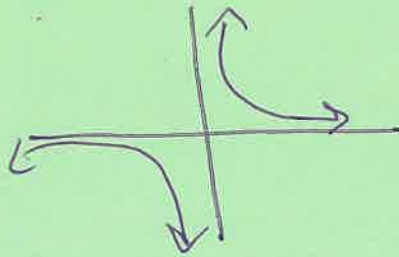
1. Let $f(x) = \dfrac{1}{x}$, $x \neq 0$.

   (a) Sketch the graph of f.

   (2)

   The graph of f is transformed to the graph of g by a translation of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

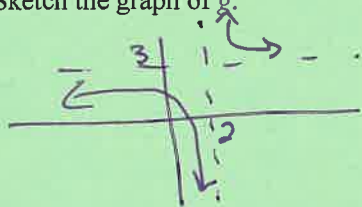   (b) Find an expression for $g(x)$. $g(x) = \dfrac{1}{x-2} + 3$

   (2)

   (c) (i) Find the intercepts of g. $g(0) = -\tfrac{1}{2} + 3 = 2\tfrac{1}{2}$ $\qquad -3 = \dfrac{1}{x-2}$

   $\qquad\qquad\qquad\qquad\qquad\qquad\qquad y = 2\tfrac{1}{2}$ $\qquad\qquad\qquad -3x + 6 = 1$

   $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad -3x = -5$

   (ii) Write down the equations of the asymptotes of g. $y = 3$ $\qquad x = \tfrac{5}{3}$

   $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad x = 2$

   (iii) Sketch the graph of g.
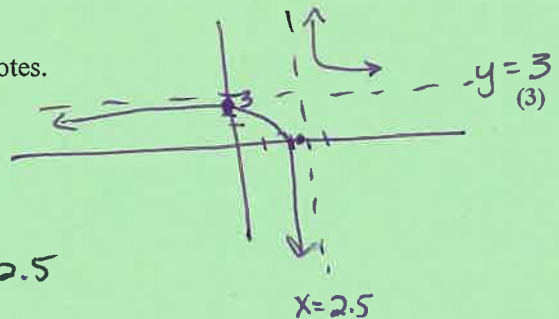

   (10)
   (Total 14 marks)

2. The function $f(x)$ is defined as $f(x) = 3 + \dfrac{1}{2x-5}$, $x \neq \dfrac{5}{2}$.

   (a) Sketch the curve of f for $-5 \leq x \leq 5$, showing the asymptotes.

   $2x - 5 = 0$

   $x = \tfrac{5}{2}$

   $y = 3$

   (3)

   (b) Using your sketch, write down

   (i) the equation of each asymptote; $y = 3$, $x = 2.5$

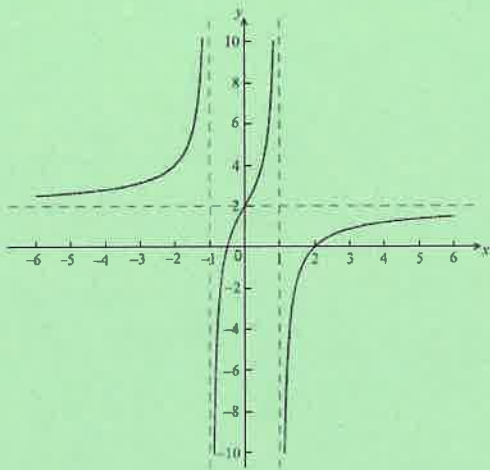   (ii) the value of the x-intercept; $y = 2.\overline{33}$

   $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad x = 2.5$

   (iii) the value of the y-intercept. $2.8 = y$

   (4)

   (Total 7 marks)

3. Let $f(x) = p - \dfrac{3x}{x^2 - q^2}$, where $p, q \in \mathbb{R}^+$.

Part of the graph of f, including the asymptotes, is shown below.

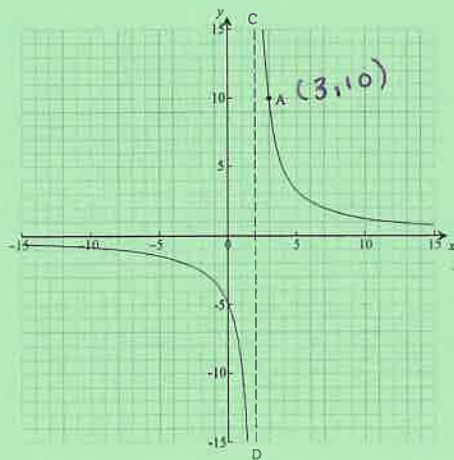(a) The equations of the asymptotes are $x = 1$, $x = -1$, $y = 2$. Write down the value of

(i) p; **2**

(ii) q. **1**   since $(x^2 - 1^2) = x^2 - 1 = (x+1)(x-1)$

(2)

(Total 2 marks)

4. (a) The diagram shows part of the graph of the function $f(x) = \dfrac{q}{x - p}$. The curve passes through the point A (3, 10). The line (CD) is an asymptote.
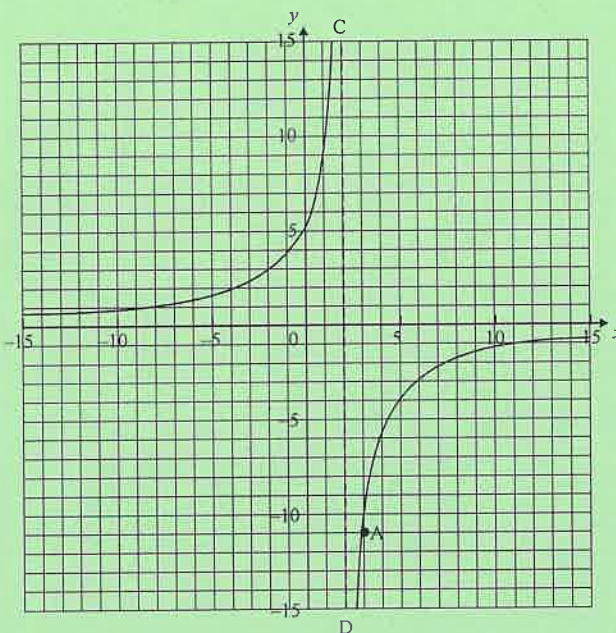

Find the value of

(i) p; **2** $10 = \dfrac{q}{3-2}$

(ii) q. **10**

                       $10 = q$

(b)   The graph of $f(x)$ is transformed as shown in the following diagram. The point A is transformed to A' $(3, -10)$.
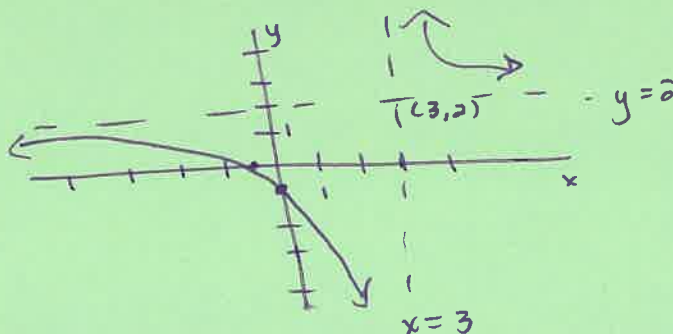

Give a full geometric description of the transformation.   *vertical reflection across the x-axis*

(Total 6 marks)

5.   The function f is given by

$$f(x) = \frac{2x+1}{x-3}, x \in \mathbb{R}, x \neq 3.$$

(a)   (i)   Show that $y = 2$ is an asymptote of the graph of $y = f(x)$.   *Since num is same degree as denom, horiz asym at $\frac{2}{1}$ or $y = 2$*

(2)

(ii)   Find the vertical asymptote of the graph.   *$x = 3$*

(1)

(iii)   Write down the coordinates of the point P at which the asymptotes intersect.   *$(3, 2)$*

(1)

(b)   Find the points of intersection of the graph and the axes   *$y = f(0) = \frac{2(0)+1}{(0)-3} = -\frac{1}{3}$*

x: $0 = 2x+1$ $x = -\frac{1}{2}$

(4)

(c)   Hence sketch the graph of $y = f(x)$, showing the asymptotes by dotted lines.

(4)

(Total 12 marks)