

Key

EUA5 Practice 2

1a. [2 marks]

Let  $f(x) = \frac{3x}{x-q}$ , where  $x \neq q$ .

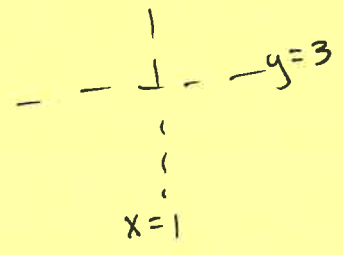
Write down the equations of the vertical and horizontal asymptotes of the graph of  $f$ .

~~vertical asymptote~~  
~~horizontal asymptote~~

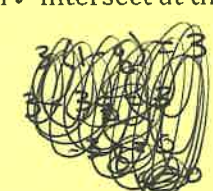
1b. [2 marks]

$x = q$   
 $y = \frac{3}{1} = 3$

The vertical and horizontal asymptotes to the graph of  $f$  intersect at the point  $Q(1, 3)$ .



Find the value of  $q$ .



$Q(1, 3)$   
↑  
vertical asymptote

1c. [4 marks]

The vertical and horizontal asymptotes to the graph of  $f$  intersect at the point  $Q(1, 3)$ .

$PQ = \sqrt{(x-1)^2 + (\frac{3x}{x-1} - 3)^2}$   
 $= \sqrt{(x-1)^2 + (\frac{3x - 3(x-1)}{x-1})^2}$   
 $= \sqrt{(x-1)^2 + (\frac{3x - 3x + 3}{x-1})^2}$   
 $= \sqrt{(x-1)^2 + (\frac{3}{x-1})^2}$

The point  $P(x, y)$  lies on the graph of  $f$ . Show that  $PQ = \sqrt{(x-1)^2 + (\frac{3}{x-1})^2}$ .

2a. [2 marks]

Let  $f(x) = 3x - 2$  and  $g(x) = \frac{5}{3x}$ , for  $x \neq 0$ .

Find  $f^{-1}(x)$ .  
 $x = 3y - 2$   
 $x + 2 = 3y$   
 $y = \frac{x+2}{3} = f^{-1}(x)$

2b. [2 marks]

Show that  $(g \circ f^{-1})(x) = \frac{5}{x+2}$ .  
 $g(f^{-1}(x)) = g(\frac{x+2}{3}) = \frac{5}{3(\frac{x+2}{3})} = \frac{5}{x+2}$

2c. [2 marks]

Let  $h(x) = \frac{5}{x+2}$ , for  $x \geq 0$ . The graph of  $h$  has a horizontal asymptote at  $y = 0$ .

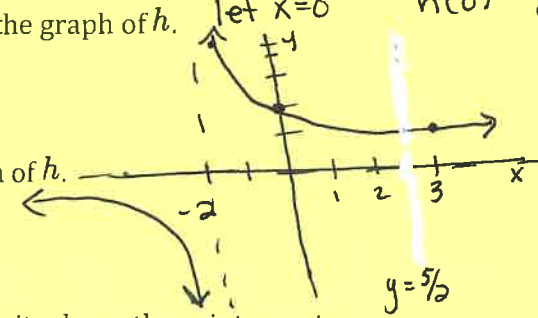
vertical asymptote at  $x = -2$

Find the  $y$ -intercept of the graph of  $h$ .

let  $x=0$   
 $h(0) = \frac{5}{0+2} = \frac{5}{2} = y$

2d. [3 marks]

Hence, sketch the graph of  $h$ .



$h(2) = \frac{5}{3+2} = \frac{5}{5} = 1 \Rightarrow (3, 1)$

$h(-1) = \frac{5}{-1+2} = \frac{5}{1} = 5$

$h(-3) = \frac{5}{-3+2} = \frac{5}{-1} = -5$

2e. [1 mark]

For the graph of  $h^{-1}$ , write down the  $x$ -intercept;

~~4/2~~  
 $y = 5/2$  on  $h(x)$   
 $x = 5/2$  on  $h^{-1}(x)$

2f. [1 mark]

For the graph of  $h^{-1}$ , write down the equation of the vertical asymptote.  $x = 0$

2g. [3 marks]

Given that  $h^{-1}(a) = 3$ , find the value of  $a$ .  $(1, 3)$  since  $h(3) = 1$ ,  $h^{-1}(1) = 3$   $h^{-1} \rightarrow (1, 3)$   
 $a = 1$

3a. [1 mark]

Let  $f(x) = p + \frac{9}{x-q}$ , for  $x \neq q$ . The line  $x = 3$  is a vertical asymptote to the graph of  $f$ .

Write down the value of  $q$ .  $q = 3$

3b. [4 marks]

The graph of  $f$  has a  $y$ -intercept at  $(0, 4)$ .

$$4 = p + \frac{9}{0-3}$$

Find the value of  $p$ .

$$4 = p + \frac{9}{-3}$$

$$4 = p - 3$$

$$7 = p$$

3c. [1 mark]

The graph of  $f$  has a  $y$ -intercept at  $(0, 4)$ .

Write down the equation of the horizontal asymptote of the graph of  $f$ .

$$y = 0$$