EUA5 Practice 2

1a. *[2 marks]*

Let $f(x) = \frac{3x}{x-q}$, where $x \neq q$.

Write down the equations of the vertical and horizontal asymptotes of the graph of f. $\boxed{x = q}$ $\boxed{y = 3/1 = 3}$

1b. *[2 marks]*

The vertical and horizontal asymptotes to the graph of f intersect at the point $\mathrm{Q}(1, 3)$.

x=1 vert asymp. y=3 horiz. asymptote

Find the value of q. $\boxed{q = 1}$

1c. *[4 marks]*

The vertical and horizontal asymptotes to the graph of f intersect at the point $\mathrm{Q}(1, 3)$.

The point $\mathrm{P}(x,\ y)$ lies on the graph of f. Show that $\mathrm{PQ} = \sqrt{(x-1)^2 + \left(\frac{3}{x-1}\right)^2}$.

2a. *[2 marks]*

Let $f(x) = 3x - 2$ and $g(x) = \frac{5}{3x}$, for $x \neq 0$.

Find $f^{-1}(x)$.

$x = 3y - 2$

$3y = x + 2$

$\boxed{y = \frac{x+2}{3} = f^{-1}(x)}$

2b. *[2 marks]*

Show that $(g \circ f^{-1})(x) = \frac{5}{x+2}$.

$g\left(\frac{x+2}{3}\right) = \frac{5}{3\left(\frac{x+2}{3}\right)} = \frac{5}{x+2} = (g \circ f^{-1})(x)$

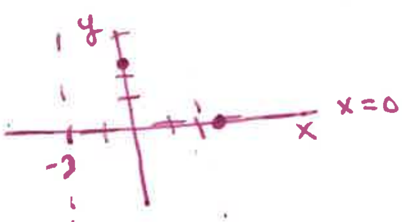
2c. *[2 marks]*

Let $h(x) = \frac{5}{x+2}$, for $x \geqslant 0$. The graph of h has a horizontal asymptote at $y = 0$.

Find the y-intercept of the graph of h. $h(0) = \frac{5}{0+2} = 5/2$

2d. *[3 marks]*

Hence, sketch the graph of h.



2e. *[1 mark]*

For the graph of h^{-1}, write down the x-intercept;

$x = \frac{5}{y+2}$

$(y+2)x = 5$

$y + 2 = \frac{5}{x}$

$y = \frac{5}{x} - 2 = h^{-1}(x)$

$0 = \frac{5}{x} - 2$

$2 = \frac{5}{x}$

$2x = 5$

$\boxed{x = 5/2}$

2f. *[1 mark]*

For the graph of h^{-1}, write down the equation of the vertical asymptote.

$h^{-1}(x) = \frac{5}{x} - 2$ or $\frac{5-2x}{x}$

$\boxed{x = 0 \text{ is the asymptote}}$

2g. *[3 marks]*

Given that $h^{-1}(a) = 3$, find the value of a.

$3 = \frac{5}{a} - 2$

$5 = \frac{5}{a}$

$5 = 5 \Rightarrow \boxed{a = 1}$

3a. *[1 mark]*

Let $f(x) = p + \frac{9}{x-q}$, for $x \neq q$. The line $x = 3$ is a vertical asymptote to the graph of f.

Write down the value of q.

$\boxed{q = 3}$

3b. *[4 marks]*

The graph of f has a y-intercept at $(0, 4)$.

x $f(x)$

Find the value of p.

$4 = p + \frac{9}{0-3}$

$4 = p - 3$

$\boxed{7 = p}$

3c. *[1 mark]*

The graph of f has a y-intercept at $(0, 4)$.

Write down the equation of the horizontal asymptote of the graph of f.

$f(x) = 7 + \frac{9}{x-3}$

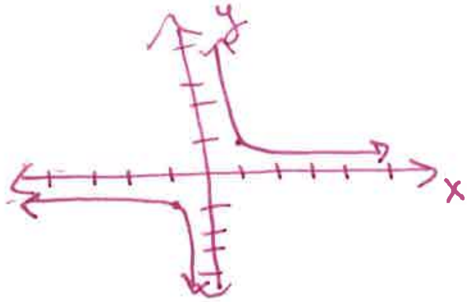
↑

vertical shift up 7

horizontal asymptote at $\boxed{y = 7}$

1. Let $f(x) = \frac{1}{x}$, $x \neq 0$.

(a) Sketch the graph of f. **(2)**



The graph of f is transformed to the graph of g by a translation of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Right 2 up 3

(b) Find an expression for $g(x)$. $g(x) = \frac{1}{x-2} + 3$ **(2)**

(c) (i) Find the intercepts of g.

$0 = \frac{1}{x-2} + 3$

$-3 = \frac{1}{x-2}$

$-3(x-2) = 1$

$-3x+6 = 1$

$-3x = -5$

$\boxed{x = 5/3}$

$g(0) = \frac{1}{0-2} + 3$

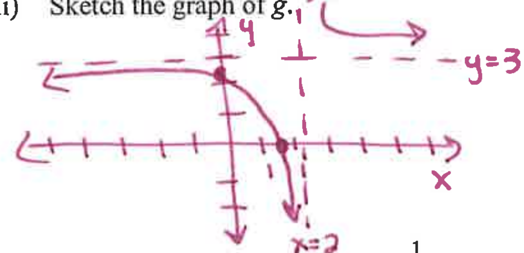
$= -\frac{1}{2} + 3$

$\boxed{g(0) = 5/2}$

$\boxed{(5/3, 0) \text{ and } (0, 5/2)}$

(ii) Write down the equations of the asymptotes of g. $\boxed{y = 3,\ x = 2}$

(iii) Sketch the graph of g. **(10)**



(Total 14 marks)

2. The function $f(x)$ is defined as $f(x) = 3 + \frac{1}{2x-5}$, $x \neq \frac{5}{2}$.

(a) Sketch the curve of f for $\boxed{-5 \leq x \leq 5}$, showing the asymptotes. **(3)**

$f(0) = 3 + \frac{1}{-5} = 2.8$

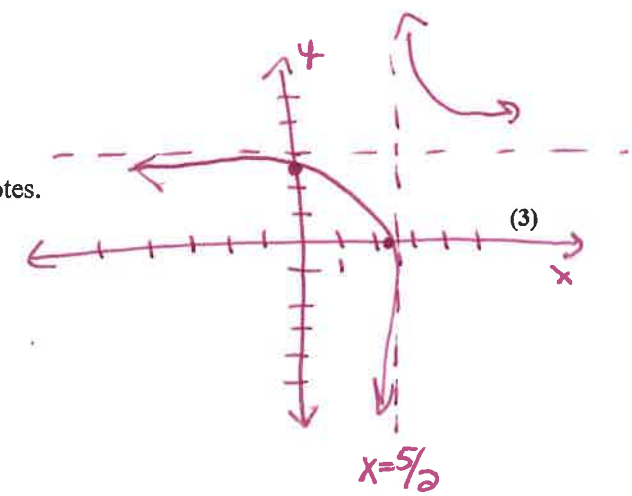
$0 = 3 + \frac{1}{2x-5}$

$-3 = \frac{1}{2x-5}$

$-6x + 15 = 1$ $-6x = -14$

$x = 14/6$

$x \approx 2.3$



(b) Using your sketch, write down

(i) the equation of each asymptote; $\boxed{y = 3,\ x = 5/2}$

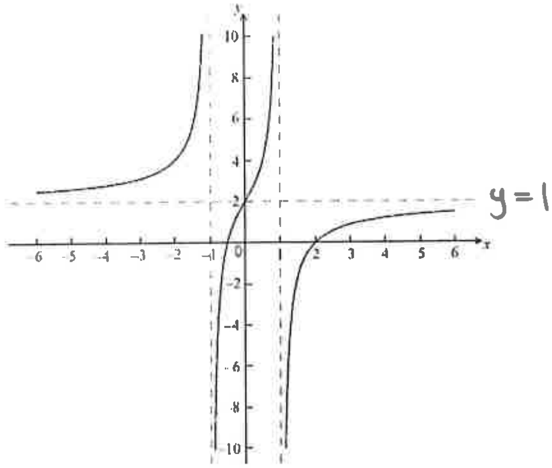
(ii) the value of the x-intercept; $\boxed{(2.3, 0)}$

(iii) the value of the y-intercept. $\boxed{(0, 2.8)}$ **(4)**

(Total 7 marks)

3. Let $f(x) = p - \frac{3x}{x^2 - q^2}$, where $p, q \in \mathbb{R}^+$.

Part of the graph of f, including the asymptotes, is shown below.



(a) The equations of the asymptotes are $x = 1, x = -1, y = 2$. Write down the value of

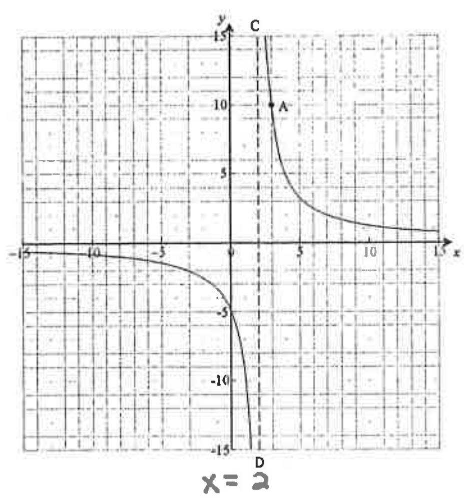
(i) p; 2

(ii) q. 1

(2)

(Total 2 marks)

4. (a) The diagram shows part of the graph of the function $f(x) = \frac{q}{x-p}$. The curve passes through the point A (3, 10). The line (CD) is an asymptote.



Find the value of

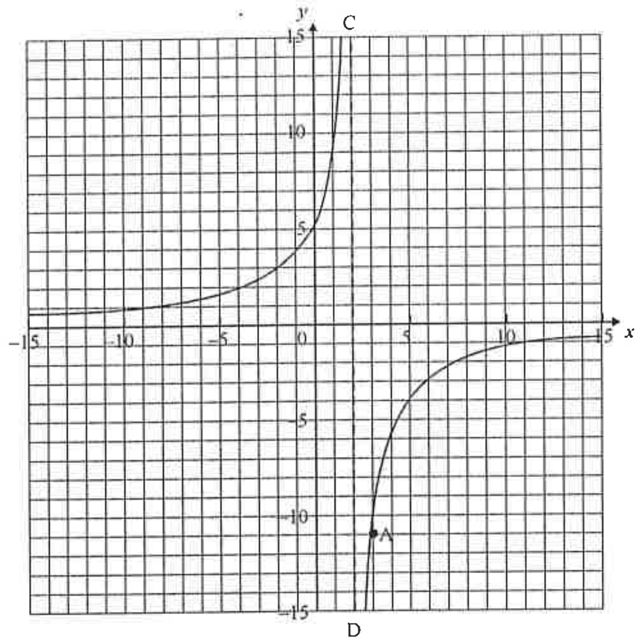
(i) p; 2

(ii) q. 10

$10 = \frac{q}{3-2}$

$10 = q$

(b) The graph of $f(x)$ is transformed as shown in the following diagram. The point A is transformed to A′ $(3, -10)$.



$(x, y) \rightarrow (x, -y)$

Give a full geometric description of the transformation.

f(x) is vertically reflected over the x-axis

(Total 6 marks)

5. The function f is given by

$$f(x) = \frac{2x+1}{x-3}, x \in \mathbb{R}, x \neq 3.$$

(a) (i) Show that $y = 2$ is an asymptote of the graph of $y = f(x)$. Since $\frac{2\cancel{x}}{\cancel{x}}$ same degree, the horizontal asymptote is $y = 2/1 = 2$ **(2)**

(ii) Find the vertical asymptote of the graph.

$x = 3$ **(1)**

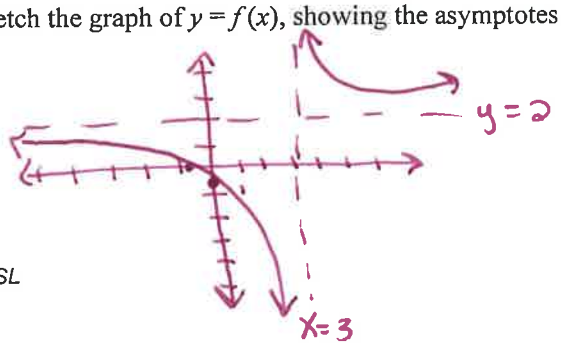
(iii) Write down the coordinates of the point P at which the asymptotes intersect. **(1)**

$(3, 2)$

(b) Find the points of intersection of the graph and the axes. $f(0) = \frac{2(0)+1}{(0)-3} = -\frac{1}{3}$ $\boxed{(0, -\frac{1}{3})}$

$0 = 2x + 1$ $2x = -1$ $x = -1/2$ $\boxed{(-1/2, 0)}$ **(4)**

(c) Hence sketch the graph of $y = f(x)$, showing the asymptotes by dotted lines. **(4)**



(Total 12 marks)