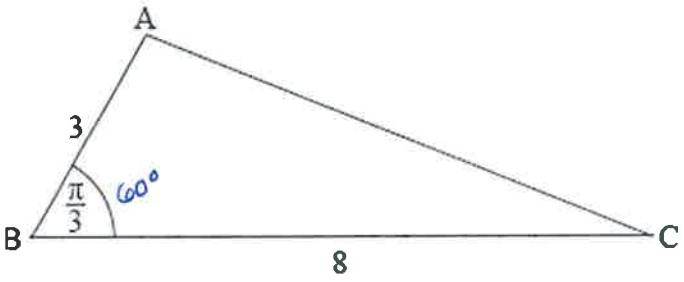
Triangle Trig Quiz 2019
EUA REVIEW

Name Key Date Period

**1.** *[4 marks]*

The following diagram shows triangle ABC, with $\mathrm{AB} = 3$ cm, $\mathrm{BC} = 8$ cm, and $\hat{\mathrm{ABC}} = \frac{\pi}{3}$.

**diagram not to scale**




Cosines

$AC = \sqrt{(3)^2 + (8)^2 - 2(3)(8)\cos(60°)}$

$= \sqrt{9 + 64 - 48(\frac{1}{2})}$

$= \sqrt{73 - 24}$

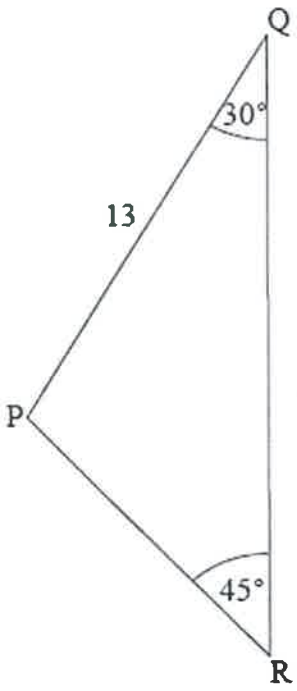
$= \sqrt{49} = 7\text{cm}.$

Show that $\mathrm{AC} = 7$ cm.

**2.** *[6 marks]*

The following diagram shows triangle PQR.

**diagram not to scale**




$\frac{\sin 30}{PR} = \frac{\sin 45}{13}$

$13 \sin 30 = PR \sin 45$

$\frac{13 \sin 30}{\sin 45} = PR$

$\frac{13(\frac{1}{2})}{\frac{\sqrt{2}}{2}} = \frac{\frac{13}{2}}{\frac{\sqrt{2}}{2}} = \frac{13}{\sqrt{2}} = \frac{13\sqrt{2}}{2} = PR$

$\hat{\mathrm{PQR}} = 30°$, $\hat{\mathrm{QRP}} = 45°$ and $\mathrm{PQ} = 13$ cm.

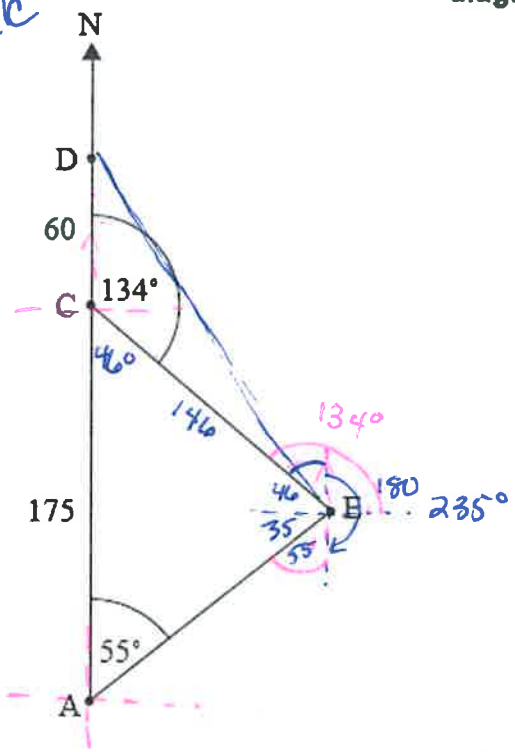
Find PR.

**3a.** *[2 marks]*

A ship is sailing north from a point A towards point D. Point C is 175 km north of A. Point D is 60 km north of C. There is an island at E. The bearing of E from A is 055°. The bearing of E from C is 134°. This is shown in the following diagram.

Calc

**diagram not to scale**




Find the bearing of A from E. 235°

**3b.** *[5 marks]* $\frac{\sin 55}{CE} = \frac{\sin 79}{175}$ $\frac{175 \sin 55}{\sin 79} = CE \approx 146$ Km

Finds CE.

**3c.** *[3 marks]*

Find DE. $DE = \sqrt{60^2 + 146^2 - 2(60)(146)\cos 134^\circ} \approx 193$ Km

**4a.** *[3 marks]*

The following diagram shows triangle ABC.

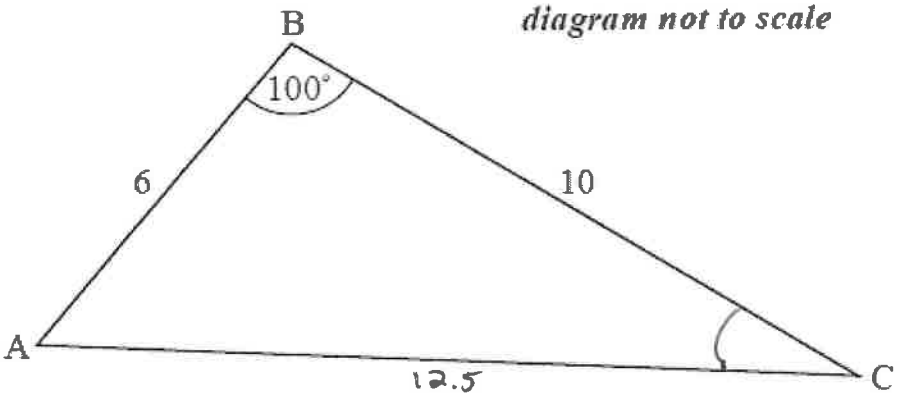



*diagram not to scale*

$AB = 6\,\text{cm}$, $BC = 10\,\text{cm}$, and $A\hat{B}C = 100°$.

Find AC. $AC = \sqrt{6^2+10^2-2(6)(10)\cos 100} \approx 12.5\,\text{cm}$

**4b.** *[3 marks]*

Find $B\hat{C}A$. $\frac{\sin C}{6} = \frac{\sin 100}{12.5}$ $\quad C = \sin^{-1}\left(\frac{6\sin 100}{12.5}\right) \approx 28.2°$

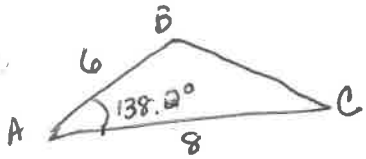
**5a.** *[4 marks]*

In triangle ABC, $AB = 6\,\text{cm}$ and $AC = 8\,\text{cm}$. The area of the triangle is $16\,\text{cm}^2$.

Find the two possible values for $\hat{A}$.

1) 41.8°

2) 138.2°




$A = \frac{1}{2}ab\sin A$

$16 = \frac{1}{2}(6)(8)\sin A$

$\sin^{-1}\left(\frac{16}{24}\right) = A$ $\quad A \approx 41.8°$

**5b.** *[3 marks]*

Given that $\hat{A}$ is obtuse, find BC.

$BC = \sqrt{6^2+8^2-2(6)(8)\cos 138.2°}$

$\approx 13.1\,\text{cm}$

**6a.** *[1 mark]*

The diagram below shows a quadrilateral ABCD with obtuse angles $A\hat{B}C$ and $A\hat{D}C$.

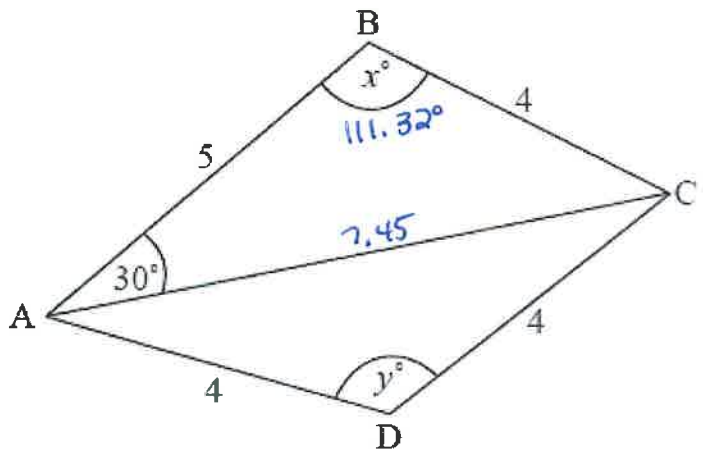



*diagram not to scale*

Cale.

AB = 5 cm, BC = 4 cm, CD = 4 cm, AD = 4 cm, $B\hat{A}C = 30°$, $A\hat{B}C = x°$, $A\hat{D}C = y°$.

Use the cosine rule to show that $AC = \sqrt{41 - 40\cos x}$.

$AC = \sqrt{5^2+4^2-2(5)(4)\cos x}$
$= \sqrt{25+16-40\cos x}$
$= \sqrt{41-40\cos x}$

**6b.** *[2 marks]*

Use the sine rule in triangle ABC to find another expression for AC.

$\frac{\sin 30°}{4} = \frac{\sin x}{AC}$ $\quad 4\sin x = AC \sin 30$ $\quad \frac{4\sin x}{\sin 30} = AC$ $\quad \frac{4\sin x}{\frac{1}{2}} = AC$ $\quad AC = 8\sin x$

**6c.** *[6 marks]*

(i) Hence, find $x$, giving your answer to two decimal places.

$(\sqrt{41-40\cos x})^2 = (8\sin x)^2$
$41 - 40\cos x = 64\sin x$

Sketch, showing intersection
Find $x \approx 8.68$
$111.32°$

(ii) Find AC.

$AC = 8\sin(111.32) \approx 7.45$

**6d.** *[5 marks]*

(i) Find $y$. $y = \cos^{-1}\left(\frac{4^2+4^2-7.45^2}{2(4)(4)}\right) \approx 137°$

(ii) Hence, or otherwise, find the area of triangle ACD.

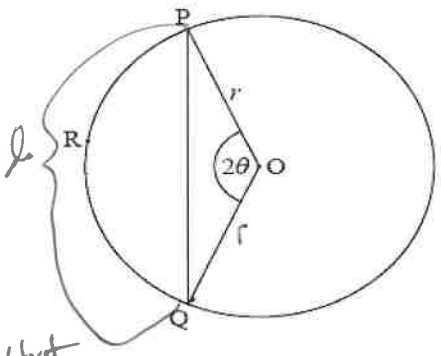
$A = \frac{1}{2}ab\sin C$ $\quad A = \frac{1}{2}(4)(4)\sin(137°)$
$A \approx 5.45\text{ cm}^2$

Key

# Sectors and Arcs Review 2019

**1a.** *[4 marks]*

Consider the following circle with centre O and radius $r$.




need double-angle formula!

Given that

The points P, R and Q are on the circumference, $\hat{PQ} = 2\theta$, for $0 < \theta < \frac{\pi}{2}$.

Use the cosine rule to show that $PQ = 2r \sin\theta$.

$PQ = \sqrt{r^2 + r^2 - 2r^2\cos(2\theta)}$
$= \sqrt{2r^2 - 2r^2(\cos(2\theta))}$
$= \sqrt{2r^2(1 - \cos(2\theta))}$
$= \sqrt{2r^2(1 - (1 - \sin^2\theta))}$
$= \sqrt{2r^2(\sin^2\theta)} = 2r\sin\theta$

**1b.** *[5 marks]*

only do this Given that $PQ = 2r\sin\theta$

Let $l$ be the length of the arc PRQ.

Given that $1.3PQ - l = 0$, find the value of $\theta$.

$1.3(2r\sin\theta) - r2\theta = 0$
$2.6r\sin\theta = r2\theta$

θ | sinθ
(1.22, 2.44)

graph

**1c.** *[4 marks]*

Consider the function $f(\theta) = 2.6\sin\theta - 2\theta$, for $0 < \theta < \frac{\pi}{2}$.

(i) Sketch the graph of $f$.

(ii) Write down the root of $f(\theta) = 0$.

**1d.** *[3 marks]*

Use the graph of $f$ to find the values of $\theta$ for which $l < 1.3PQ$.

**2a.** *[2 marks]*

The diagram below shows a circle with centre O and radius 8 cm.

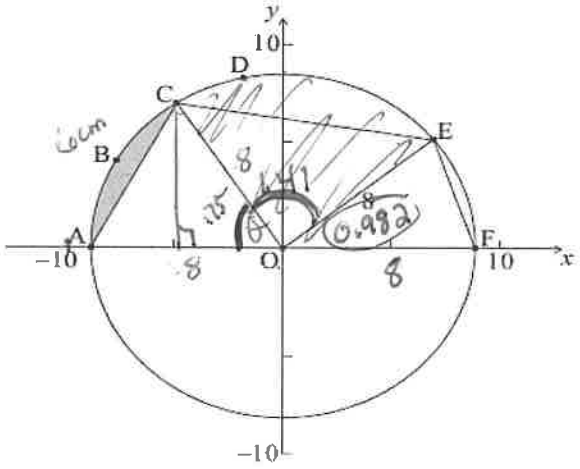



*diagram not to scale*

The points A, B, C, D, E and F are on the circle, and [AF] is a diameter. The length of arc ABC is 6 cm.

Find the size of angle AOC. $s = r\theta$

$6 = 8\theta$

$\theta = \frac{6}{8}$ rad. $= \frac{3}{4}$ rad

**2b.** *[6 marks]*

Hence find the area of the shaded region.

Sector area: $\frac{1}{2}r^2\theta = \frac{1}{2}(8)^2(\frac{3}{4}) = 24$

Area of $\Delta = \frac{1}{2}ab\sin\theta$

$= \frac{1}{2}(8)(8)\sin(\frac{3}{4})$

$= 21.8$ cm

$24.0 - 21.8 = 2.14$

**2c.** *[2 marks]*

The area of sector OCDE is 45 $cm^2$.

$45 = \frac{1}{2}(8)^2$ (COE)

$\frac{45}{32} = 1.41$

Find the size of angle COE.

**2d.** *[5 marks]*

$\pi - 1.41 - .75 = 0.982$

Find EF.

$EF = \sqrt{8^2 + 8^2 - 2(8)(8)\cos(0.982)}$

$EF \approx 7.54$ cm

**3a.** *[3 marks]*

The following diagram shows a circle with centre O and radius 4 cm.

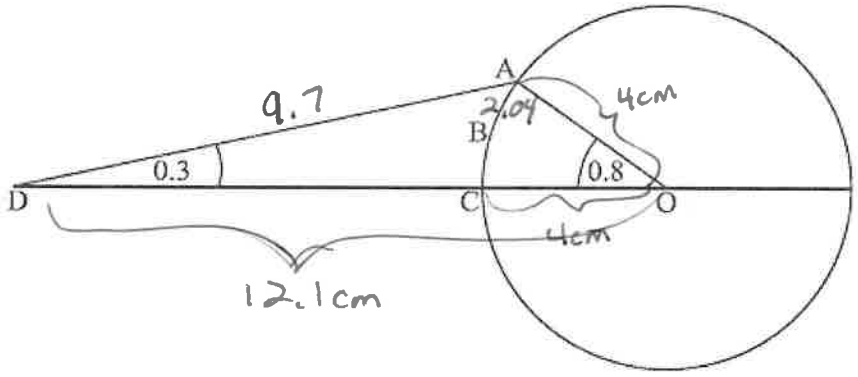



*diagram not to scale*

The points A, B and C lie on the circle. The point D is outside the circle, on (OC).

Angle ADC = 0.3 radians and angle AOC = 0.8 radians.

Find AD. AAS $\frac{\sin 0.3}{4} = \frac{\sin 0.8}{AO}$ $AO = \frac{4\sin 0.8}{\sin 0.3} = 9.7\text{cm}$

**3b.** *[4 marks]*

Find OD. $\pi - 0.8 - 0.3 = 2.04$ $\frac{\sin 2.04}{OD} = \frac{\sin 0.3}{4}$ $OD = \frac{4\sin 2.04}{\sin 0.3} \approx 12.1\text{cm}$

**3c.** *[2 marks]*

Find the area of sector OABC. $A = \frac{1}{2}r^2\theta = \frac{1}{2}(4)^2(0.8) \approx 6.4\text{cm}^2$

**3d.** *[4 marks]*

Find the area of region ABCD.

Area of $\triangle = \frac{1}{2}(a)(b)\sin C$

$= \frac{1}{2}(9.7)(4)\sin(2.04)$

$= 17.3\text{cm}^2$

Area of ABCD $= 17.3 - 6.4$

$10.93\text{cm}^2$