4). $f(x) = xe^x$

$f'(x) = xe^x(1) + (1)(e^x)$

$= xe^x + e^x$

$= e^x(x+1) = 0$

$e^x = 0$ or $x+1 = 0$

DNE $\qquad x = -1$

$f''(x) = e^x(1) + xe^x(x+1)$

$= e^x(1 + 1(x+1))$

$= e^x(1+x+1) = e^x(2+x)$

$f''(-1) = e^{-1}(2-1) = \frac{1}{e}(1) = \frac{1}{e}$ (+)

min

relative min at $(-1, \frac{-1}{e})$

Absolute or global extrema

- the greatest and least values of a function over its entire domain

("relative" extrema occur between the endpoints on an open interval)

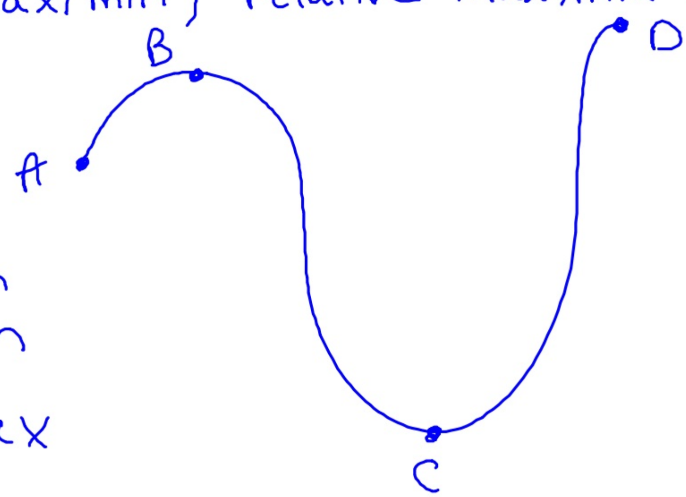
Ex| Identify the points as being absolute max/min, relative max/min or neither

A. neither

B. relative max

C. relative min
absolute min

D. absolute max



Ex] Find the absolute max/min for $f(x) = x^2 - 2x$ on $-1 \le x \le 2$

Strategy: use 1st der. test to find max/min **and** test the endpoints!

HW 7W p244 #1-4

$$f'(x) = 2x - 2$$
$$2x - 2 = 0$$
$$2x = 2$$
$$x = 1 \Longrightarrow \text{relative max or min}$$

Test endpoints and $x=1$ in $f(x)$

$f(-1) = (-1)^2 - 2(-1) = 3$ biggest

$f(1) = -1$ smallest

$f(2) = 0$ neither

$\therefore$ absolute min is -1 and abs. max is 3

OPTIMIZATION PROBLEMS

Real World applications!

- minimize cost (maximize profit)
- maximize area

Steps for solving optimization problem

1. Assign variables and create a sketch of the situation, if possible.
2. Write an equation in terms of two variables.
3. Find values that are reasonable for the situation where the derivative equals zero. (find where $f'(x)=0$).
4. Verify if the value is a max or min by using the 2nd derivative test. Remember to check the values found in #3 along with endpoints!

$\boxed{Ex}$ The product of two numbers is 48. Find the two numbers so that the sum of the first number plus three times the second number is a minimum.

1. $x =$ 1st number, $y =$ 2nd number

2. $xy = 48$ $\qquad$ $x + 3y$ is a minimum

solve for y

$$y = \frac{48}{x}$$

substitute

$$S(x) = x + 3\left(\frac{48}{x}\right) = x + \frac{144}{x}$$

3. $S'(x) = 1 + \frac{x(0) - 144(1)}{x^2} = 1 - \frac{144}{x^2}$

$$1 - \frac{144}{x^2} = 0 \Rightarrow 1 = \frac{144}{x^2} \Rightarrow x^2 = 144$$

$$x = \pm 12$$

$x = 12$

4. $\cancel{S''(x)=0}$

$$S'' = 0 - \frac{x^2(0) - 144(2x)}{(x^2)^2}$$

$$= \frac{+288x}{x^4} = \frac{+288}{x^3}$$

$$S''(12) = \frac{+288}{(12)^3} = (+)$$

minimum

~~minumum~~

min at $x = 12$

if $x=12$ is 1st, then $12y = 48 \Rightarrow y = 4$

the two numbers are 12 and 4

For Tuesday - in class

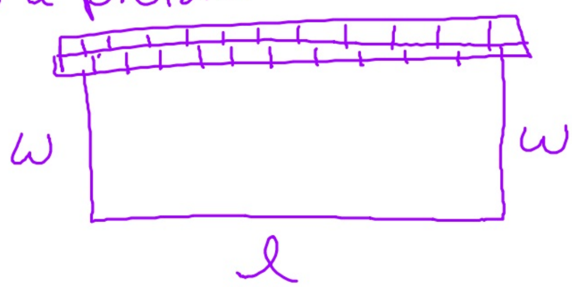
copy Example 30 page 245 into your notes

Hw 7X p. 246 #1-3.

due wed

A rectangular plot of farmland is enclosed by 180m of fencing material on 3 sides. The fourth side is bordered by a stone wall. Find the dimensions of the plot that encloses the maximum area. Find the max area.

1. Draw a picture



Know

$P = 2w + l = 180\text{m}$

$A = lw$ max

2. solve $2w + l = 180$ for l

substitute into Area function

$l = 180 - 2w$

$A(w) = (180 - 2w)w$

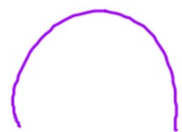
$= 180w - 2w^2$

$x = \frac{-b}{2a}$

3. $A'(w) = 180 - 4w$

$$180 - 4w = 0$$
$$-4w = -180$$
$$w = 45$$

$$w = \frac{-b}{2a} = \frac{-180}{2(-2)} = 45$$

4. $A''(w) = -4$ $(-)$  max

$$l = 180 - 2w$$
$$= 180 - 2(45) = 90$$

Dimensions to maximize Area is length of 90m and width~~s~~ of 45m

Hw# 1.

$x, y \in \mathbb{R}^+$

1. $x + y = 20 \Longrightarrow x = 20 - y$

max: $x + \sqrt{y}$

2. $f(y) = 20 - y + \sqrt{y} = 20 - y + y^{1/2}$

3. $f'(y) = -1 + \frac{1}{2} y^{-1/2}$

$= -1 + \frac{1}{2\sqrt{y}} = 0$

$\frac{1}{2\sqrt{y}} = 1 \Rightarrow 1 = 2\sqrt{y} \Rightarrow \frac{1}{2} = \sqrt{y}$

$\frac{1}{4} = y$

4. $f''(y) = -\frac{1}{4} y^{-3/2} = \frac{-1}{4\sqrt{y^3}} \Longrightarrow (-)$ $\cap$ max

back substitute

into $x = 20 - \left(\frac{1}{4}\right) = 19\,3/4$

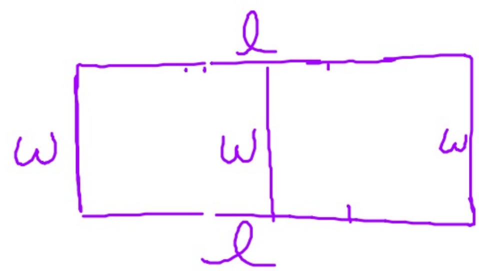
2. $a, b \in \mathbb{R}^+$

$a + 2b = 200 \implies a = 200 - 2b$

max: $a \cdot b$ or ab

$f(b) = (200 - 2b)b$

3. 400 feet of fencing



Know: $P = 400 = 3w + 2l$

$A = lw$

$3w + 2l = 400$

$2l = 400 - 3w$

$l = 200 - 3/2\,w$

$A(w) = (200 - 3/2\,w)w$

$$A(w) = (200 - \tfrac{3}{2}w)w \qquad w = \frac{-b}{2a}$$

2. $$= 200w - \tfrac{3}{2}w^2$$

3. $$A'(w) = 200 - 3w = 0$$

$$3w = 200$$

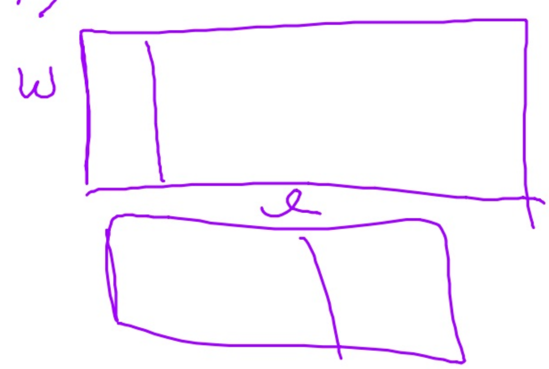
$$w = 200/3 = 66\tfrac{2}{3} \text{ ft.}$$

$$= 66.7 \text{ ft.}$$

4. $A''(w) = -3$ $(-)$ $\cap$ max

$$l = 200 - \tfrac{3}{2}(66.7)$$

$$l = 100 \text{ ft.}$$



Ex) The Cost C of ordering and storing x units of a product is $C(x) = x + \frac{10000}{x}$. A delivery truck can deliver at most 200 units per order. Find the order size that will minimize the cost. $(1 \le x \le 200)$

3. $$c(x) = 1 + \frac{x(0) - 10000(1)}{x^2} = 0$$

$$\frac{10000}{x^2} = 1 \Rightarrow x^2 = 10000 \Rightarrow x = \pm 100$$

only use $+100$

4. $$C''(x) = \frac{x^2(0) - -10000(2x)}{(x^2)^2} \Rightarrow + \cup$$ min.

$C(1) = 10001$

$C(100) = 200 \longrightarrow$ 100 units minimizes cost

$C(200) = 250$

Hw 74 p.248 1,2,5