- Binomial Expansion - a review

Recall! Pascal's Triangle

$n=0$					1					
$n=1$				1		1				
$n=2$			1		2		1			
$n=3$		1		3		3		1		
$n=4$	1		4		6		4		1	

⋮

n^{th}

<u>For Polynomials</u>

$$(a+b)^n = \sum_{r=0}^{n} \left(\binom{n}{r} (a)^{n-r} (b)^r \right)$$

$$\binom{n}{r} = {}_nC_r = \frac{n!}{r!\,(n-r)!}$$

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \cdots + \binom{n}{n}a^0 b^n$$

<u>Ex)</u> Expand $(2x-5y)^3$ $\quad a = 2x$, $b = -5y$

$$= \binom{3}{0}(2x)^3(-5y)^0 + \binom{3}{1}(2x)^2(-5y)^1 +$$

$$\binom{3}{2}(2x)^1(-5y)^2 + \binom{3}{3}(2x)^0(-5y)^3$$

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \end{array}$$

$$= (1)8x^3 + (3)(4x^2)(-5y) + (3)(2x)(25y^2)$$

$$+ (1)(-125y^3)$$

$$= 8x^3 - 60x^2y + 150xy^2 - 125y^3$$

Ex) Find the x^3 term in the expansion of $(4x-1)^9$

$$x^9 \quad x^8 \quad x^7 \quad x^6 \quad x^5 \quad x^4 \quad x^3 \quad x^2$$

$$\binom{9}{0} \quad \binom{9}{1} \quad \binom{9}{2} \quad \binom{9}{3} \quad \binom{9}{4} \quad \binom{9}{5} \quad \binom{9}{6}$$

$3+6=9$

$$\binom{9}{6}(4x)^{\boxed{3}}(-1)^{\boxed{6}}$$

$$=(84)(64x^3)(1)$$

$$=5376x^3$$

$$f(x) = (x^2+3)^7$$

$$f'(x) = 7(x^2+3)^6(2x)$$

$$= 14x(x^2+3)^6$$

$$= 14x\Big[\binom{6}{0}\underbrace{(x^2)^6}_{x^{12}}(3)^0 + \cdots\cdots \binom{6}{2}\underbrace{(x^2)^2}_{x^4}(3)^4$$

$$= 14x\binom{6}{2}(x^2)^2(3)^4$$

or $\binom{6}{4}$

$$= 14x(15)(x^4)(81) = 17010x^5$$

1 5 10 10 5 1

x^5 x^4 x^3

$$(10)(8x)^3(3)^2$$