4). $f(x) = xe^x$

$f'(x) = xe^x(1) + (1)(e^x)$

$= xe^x + e^x$

$= e^x(x+1) = 0$

$e^x = 0$ or $x + 1 = 0$

DNE $\qquad x = -1$

$f''(x) = e^x(1) + xe^x(x+1)$

$= e^x(1 + 1(x+1))$

$= e^x(1 + x + 1) = e^x(2+x)$

$f''(-1) = e^{-1}(2-1) = \frac{1}{e}(1) = \frac{1}{e}$ (+)

min

relative min at $\left(-1, \frac{-1}{e}\right)$

<u>Absolute</u> or <u>global</u> extrema

- the greatest and least values of a function over its entire domain

("relative" extrema occur between the endpoints on an <u>open</u> interval)

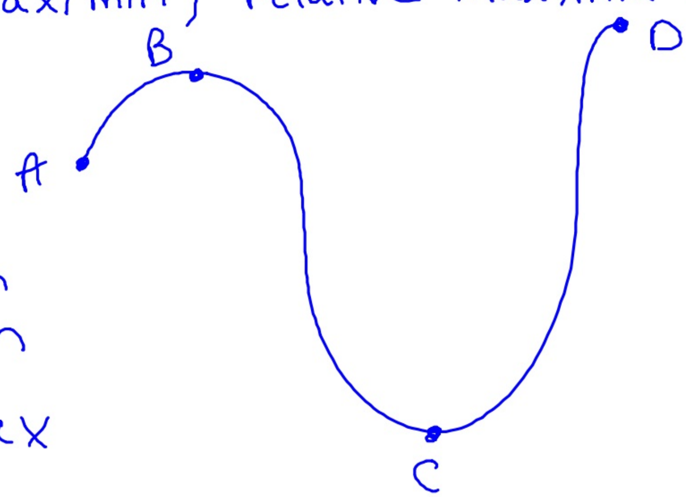
<u>Ex</u> Identify the points as being absolute max/min, relative max/min or neither

A. neither

B. relative max

C. relative min
absolute min

D. absolute max



Ex| Find the absolute max/min for $f(x) = x^2 - 2x$ on $-1 \le x \le 2$

Strategy: use 1st der. test to find max/min
and test the endpoints!

HW 7W p244 #1-4

$f'(x) = 2x - 2$

$2x - 2 = 0$

$2x = 2$

$x = 1 \Longrightarrow$ relative max or min

Test endpoints and $x = 1$ in $f(x)$

$f(-1) = (-1)^2 - 2(-1) = 3$ biggest

$f(1) = -1$ smallest

$f(2) = 0$ neither

$\therefore$ absolute min is -1 and abs. max is 3

OPTIMIZATION PROBLEMS

Real World applications!

- minimize cost (maximize profit)
- maximize area

Steps for solving optimization problem

1. Assign variables and create a sketch of the situation, if possible.
2. Write an equation in terms of two variables.
3. Find values that are reasonable for the situation where the derivative equals zero. (find where $f'(x)=0$).
4. Verify if the value is a max or min by using the 2nd derivative test. Remember to check the values found in #3 along with endpoints!

$\boxed{Ex}$ The product of two numbers is 48. Find the two numbers so that the sum of the first number plus three times the second number is a minimum.

1. $x =$ 1st number, $y =$ 2nd number

2. $xy = 48$ $\quad\quad x + 3y$ is a minimum

solve for y

$$y = \frac{48}{x}$$

substitute

$$S(x) = x + 3\left(\frac{48}{x}\right) = x + \frac{144}{x}$$

3. $$S'(x) = 1 + \frac{x(0) - 144(1)}{x^2} = 1 - \frac{144}{x^2}$$

$$1 - \frac{144}{x^2} = 0 \Rightarrow 1 = \frac{144}{x^2} \Rightarrow x^2 = 144$$

$$x = \pm 12$$

$x = 12$

4. ~~S''(x)=0~~

$$S'' = 0 - \frac{x^2(0) - 144(2x)}{(x^2)^2}$$

$$= \frac{+288x}{x^4} = \frac{+288}{x^3}$$

$$S''(12) = \frac{+288}{(12)^3} = (+)$$

minimum

~~minumum~~

min at $x = 12$

if $x=12$ is 1st, then $12y = 48 \Rightarrow y = 4$

. the two numbers are 12 and 4

For Tuesday - in class

copy Example 30 page 245 into your notes

Hw 7X p. 246 #1-3.

due wed