3) $f(x) = \dfrac{4}{x^2+3}$

$$f'(x) = \frac{(x^2+3)(0) - (4)(2x)}{(x^2+3)^2}$$

$$f'(x) = \frac{8x}{(x^2+3)^2}$$

3) $f(x) = \ln(3x^5)$

$$f'(x) = \frac{1}{3x^5} \cdot (15x^4)$$

$$= \frac{5}{x} = 5x^{-1}$$

6) $f(x) = (\ln x)^3$

$$f'(x) = 3(\ln x)^2 \left(\frac{1}{x}\right) = \frac{3(\ln x)^2}{x}$$

10) $f(x) = e^{4x^3}$

$f'(x) = 12x^2 e^{4x^3}$

$= e^{4x^3} \cdot \frac{d}{dx}(4x^3)$

$= e^{4x^3} \cdot 12x^2$

$= 12x^2 e^{4x^3}$

more chain rule examples

Ex 1) use the chain rule to find the derivative of $f(x)=\frac{1}{x^2+1}$

$$f(x) = (x^2+1)^{-1}$$

$$f'(x) = -1(x^2+1)^{-2}\cdot(2x)$$

$$= -2x(x^2+1)^{-2} = \frac{-2x}{(x^2+1)^2}$$

Ex 2 $f(x) = x\sqrt{1-x^2} = x\,\underbrace{(1-x^2)^{1/2}}_{\text{2nd}}$

Product Rule is necessary

first · der 2nd + 2nd der 1st

$$f'(x) = \underbrace{x \cdot \underbrace{\tfrac{1}{2}(1-x^2)^{-1/2}(-2x)}_{\text{chain}} + (1-x^2)^{1/2}(1)}_{\text{product}}$$

$$= -x^2(1-x^2)^{-1/2} + (1-x^2)^{1/2}$$

$$= \frac{-x^2}{\sqrt{1-x^2}} + \sqrt{1-x^2} = \frac{1-2x^2}{\sqrt{1-x^2}}$$

Ex3) $f(x) = e^{2(3x-1)^4}$

$$f'(x) = e^{2(3x-1)^4} \cdot 8(3x-1)^3 \cdot 3$$

$$= 3e^{2(3x-1)^4} \cdot 8(3x-1)^3$$

$$= 24e^{2(3x-1)^4} \cdot (3x-1)^3$$

Ex4) $f(x) = \ln\left(\dfrac{x}{x^2+1}\right)$

$$f'(x) = \frac{1}{\frac{x}{x^2+1}} \cdot \frac{(x^2+1)(1) - (x)(2x)}{(x^2+1)^2}$$

$$\frac{4}{\frac{1}{2}}$$

$$4 \cdot \frac{2}{1} = 8$$

$$= \frac{\cancel{x^2+1}}{x} \cdot \frac{x^2+1-2x^2}{(x^2+1)^{\cancel{2}}}$$

$$= \frac{1-x^2}{x(x^2+1)} =$$

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