

$$5. f(x) = \sqrt{e^{2x} + e^{-2x}} = (e^{2x} + e^{-2x})^{1/2}$$

$$f'(x) = \frac{1}{2}(e^{2x} + e^{-2x})^{-1/2} [e^{2x} \cdot 2 + e^{-2x}(-2)] \frac{d}{du} u^n = nu^{n-1}$$
$$= \frac{2e^{2x} - 2e^{-2x}}{2\sqrt{e^{2x} + e^{-2x}}} = \frac{e^{2x} - e^{-2x}}{\sqrt{e^{2x} + e^{-2x}}}$$

$$6. f(x) = \ln(1 - 2x^3)$$

$$f'(x) = \frac{1}{(1 - 2x^3)} (-6x^2) = \frac{-6x^2}{(1 - 2x^3)}$$

$$7. f(x) = \ln(\ln(x^2))$$

$$f'(x) = \frac{1}{\ln(x^2)} \cdot \frac{1}{x^2} \cdot 2x$$

$$= \frac{2}{x \ln(x^2)} = \frac{2}{x (2 \ln x)}$$

$$= \frac{1}{x \ln x}$$

$$\begin{aligned} 12) f(x) &= x^3 \ln x \\ &= x^3 \left(\frac{1}{x} \right) + \ln x (3x^2) \end{aligned}$$

$$0 = x^2 + 3x^2 \ln x$$

$$0 = x^2 (1 + 3 \ln x)$$

$$x^2 = 0 \quad \text{or} \quad 1 + 3 \ln x = 0$$

$$x = 0$$

$$3 \ln x = -1$$

$$\ln x = -\frac{1}{3}$$

$$x = e^{-\frac{1}{3}} \approx 0.717$$



$$13) f(x) = x^{-3}, g(x) = 1-2x \quad h(x) = (f \circ g)(x) \\ = f(g(x))$$

$$h = f(1-2x) = (1-2x)^{-3}$$

$$h' = -3(1-2x)^{-4}(-2)$$

$$= \frac{6}{(1-2x)^4}$$

$6 > 0$, since the denominator is raised to an even power, there exists no real value of x for

which $h(x) < 0$

Higher Order Derivatives

$f'(x) = \frac{dy}{dx}$ is called the first derivative of y with respect to x (gradient of tangent line at x)

- the 1st derivative is generally a function. As such we could find the gradient of this function at x .

$$f''(x) = \frac{d^2 y}{dx^2} \quad \text{"double prime"}$$

- we can continue until derivative is 0

$$f'(x) = \frac{dy}{dx} \quad f^{(n)} = \frac{d^n y}{dx^n}$$

$$f''(x) = \frac{d^2 y}{dx^2}$$

$$f'''(x) = \frac{d^3 y}{dx^3}$$

Ex] Find the 1st 2 derivatives of

$$f(x) = x^4 + 3x^2 + x$$

$$f'(x) = 4x^3 + 6x + 1$$

$$f''(x) = 12x^2 + 6$$

Ex] $f'(x) = \sqrt{x^2+4}$; find $f''(x)$
 $= (x^2+4)^{1/2}$

$$f''(x) = \frac{1}{2}(x^2+4)^{-1/2} \cdot 2x$$

$$= \frac{x}{\sqrt{x^2+4}}$$

Ex) $y = 4e^{2x}$ find $\frac{d^3y}{dx^3}$

$$\frac{dy}{dx} = 4e^{2x} \cdot 2 = 8e^{2x}$$

$$\frac{d^2y}{dx^2} = 8e^{2x} \cdot 2 = 16e^{2x}$$

$$\frac{d^3y}{dx^3} = 16e^{2x} \cdot 2 = 32e^{2x}$$

$$\left. \frac{d^3y}{dx^3} \right|_{x=4}$$

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