#3) $C(n) = (3+2n)e^{-3n}$

$$\frac{dC}{dn} = (3+2n)e^{-3n}(-3) + (e^{-3n})(2)$$

$$= e^{-3n}\left[(-3)(3+2n) + 2\right]$$

$$= e^{-3n}(-9-6n+2) = e^{-3n}(-7-6n)$$

$$\frac{d^2C}{dn^2} = e^{-3n}(-6) + (-7-6n)(e^{-3n})(-3)$$

$$= -6e^{-3n} - 3(e^{-3n})(-7-6n)$$

$$= -3e^{-3n}(2+(-7-6n))$$

$$= -3e^{-3n}(-5-6n)$$

4) $\frac{dy}{dx} = \frac{4}{x} = 4x^{-1}$

$\frac{d^2y}{dx^2} = -4x^{-2}$

$\frac{d^3y}{dx^3} = 8x^{-3}$

6) $R(t) = \frac{1}{2}t \ln(t^2)$ find $\left.\frac{dR}{dt}\right|_{t=-1}$

$\frac{dR}{dt} = \frac{1}{\cancel{2}}\cancel{t}\left(\frac{1}{\cancel{t^2}}\right)\left(\cancel{2}\cancel{t}\right) + \left(\frac{1}{2}\right)\left(\ln(t^2)\right)$

$\frac{dR}{dt} = 1 + \frac{1}{2}\ln(t^2)\Big|_{t=-1}$ $1 + \frac{1}{2}\ln(1)$

$= 1+0 = 1$

8) $y = e^{x} + e^{-x}$ $\qquad \dfrac{d^n y}{dx^n}$

1st $\quad \dfrac{dy}{dx} = e^{x} + e^{-x}(-1) = e^{x} - e^{-x}$

2nd $\quad \dfrac{d^2 y}{dx^2} = e^{x} - e^{-x}(-1) = e^{x} + e^{-x}$

3rd $\quad \dfrac{d^3 y}{dx^2} = e^{x} + e^{-x}(-1) = e^{x} - e^{-x}$

$\therefore$ for $e^{x} + e^{-x}$ $\dfrac{d^n y}{dx^n}$, when

n is odd, $\dfrac{d^n y}{dx^n} = e^{x} - e^{-x}$

n is even, $\dfrac{d^n y}{dx^n} = e^{x} + e^{-x}$

10) $f(x) = 3\sqrt[5]{x^2}$ is $f'(x)$ and so the

Find the gradient of the slope of the function $f(x) = 3\sqrt[5]{x^2}$

$$f(x) = 3x^{\frac{2}{5}}$$

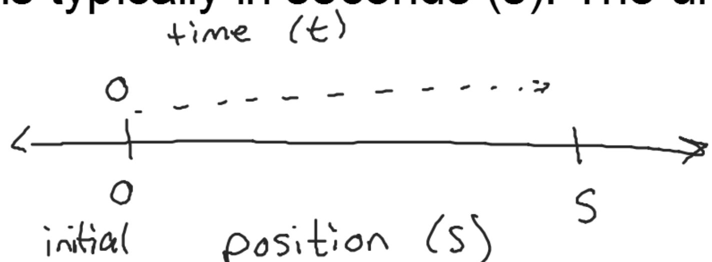
$$f'(x) = \left(\frac{2}{5}\right)3x^{\frac{2}{5}-1} = \frac{6}{5}x^{-\frac{3}{5}}$$

$$f''(x) = \left(-\frac{3}{5}\right)\frac{6}{5}x^{-\frac{3}{5}-1} = \frac{-18}{25}x^{-\frac{8}{5}}$$

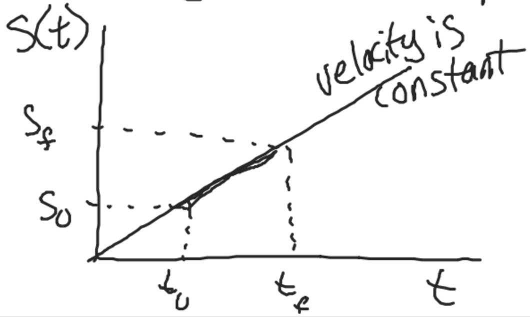
$$= \frac{-18}{25\sqrt[5]{x^8}}$$

7.5 Rates of change and motion in a line

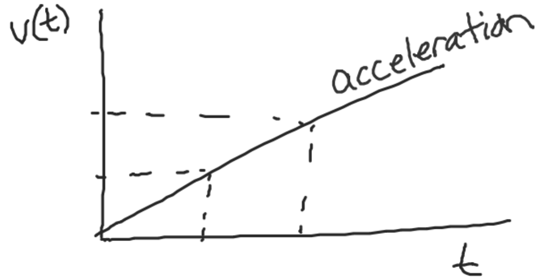
Assume a particle is moving along a straight line. Motion, in physics, is s = f(t) where s is the displacement (distance) and t is time. s is usually in meters (m) and t is typically in seconds (s). The unit is ms^{-1} $\left(\frac{m}{s}\right)$



$$\frac{\Delta s}{\Delta t} = \frac{\text{distance travelled}}{\text{time elapsed}} = \frac{f(t+\Delta t) - f(t)}{\Delta t}$$



$= \frac{\Delta y}{\Delta x}$ = slope = definition of derivative



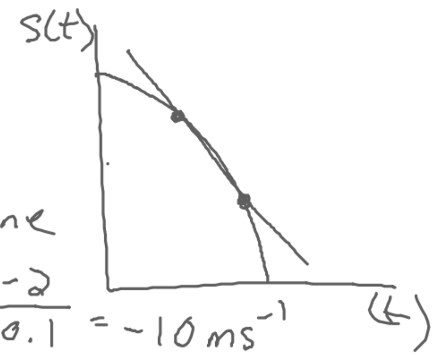
1st der gives velocity

2nd der give acceleration

3rd der gives "jerk"

4th der gives "snap"

Ex | A diver jumps from a platform at time $t=0$ sec.
The distance of the diver above the water is given by $s(t) = -4.9t^2 + 4.9t + 10$,
where s is in meters.



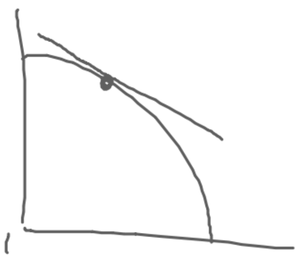
a. find <u>average</u> velocity over $[1,2]$ and $[1.1, 1]$

finding slope of <u>secant</u> line

$$\frac{\Delta S}{\Delta t} = \frac{S(1.1) - S(1)}{1.1 - 1} = \frac{1-2}{0.1} = -10\ ms^{-1}$$

b. Instantaneous velocity

the slope of the line tangent to the curve at a given $t = 1$



$v(t) = s'(t)$

$= -9.8t + 4.9$

$v(1) = -9.8(1) + 4.9 = -4.9\,ms^{-1}$

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