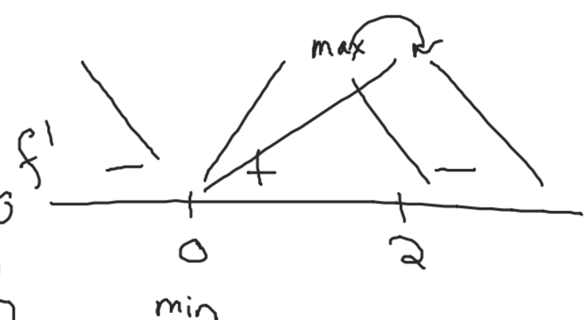
7R #6 Find the relative extrema $\qquad f'(x)=0$

$$f(x)=x^2e^{-x}$$

$$f'(x) = x^2(e^{-x})(-1) + e^{-x}(2x)$$

$$= -x^2e^{-x} + 2xe^{-x}$$

$$0 = xe^{-x}(-x+2)$$



$xe^{-x}=0 \quad$ and $\quad -x+2=0$

$$-x=-2$$

$$\boxed{x=2}$$

$\boxed{x=0}$ $\qquad \cancel{e^{-x}=0}$ $\qquad \cancel{\frac{1}{e^x}=0}$

$(0,0)$

$$f(2)=2^2e^{-2}$$

$$=\frac{4}{e^2}$$

$$(2,e^2)$$

75 #6 $f(x) = \frac{1}{x^2+1} = 1(x^2+1)^{-1}$

$$f'(x) = -1(x^2+1)^{-2}(2x)$$

$$= -2x(x^2+1)^{-2}$$

$$f''(x) = +2x\left[-2(x^2+1)^{-3}(2x)\right] - 2(x^2+1)^{-2}$$

$$= \frac{-8x^2}{(x^2+1)^3} + \frac{2}{(x^2+1)^2} \cdot \frac{(x^2+1)}{(x^2+1)}$$

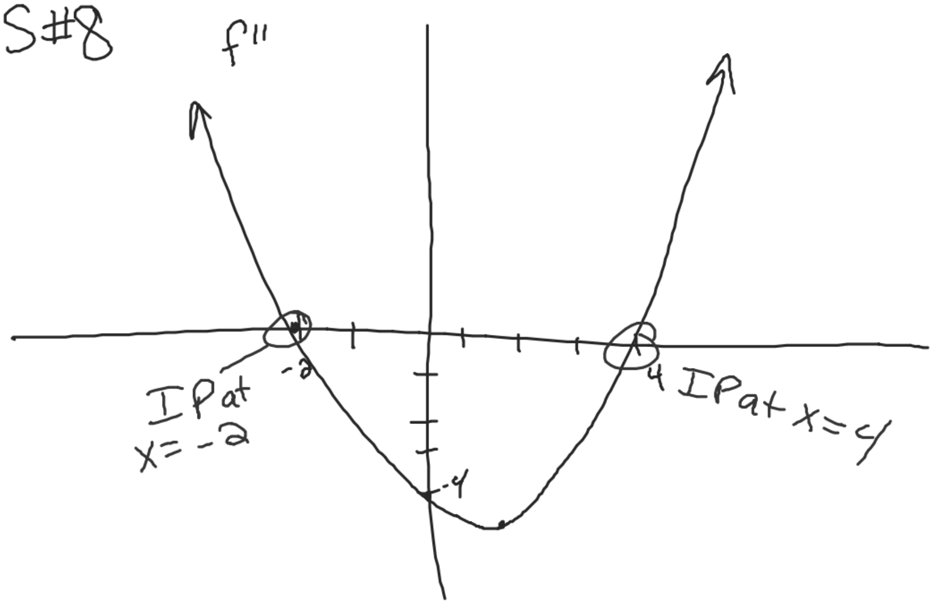
$$\frac{3x^2 - 2(x^2+1)}{(x^2+1)^3} = \frac{6x^2+2}{(x^2+1)^3}$$

$$\frac{6x^2+2}{\cancel{(x^2+1)^3}} = 0 \qquad 6x^2+2=0$$

$$6x^2 = -2$$

$$x^2 = -\frac{2}{6} = -\frac{1}{3}$$

7S#8



$f''(x) > 0$, f is concave up

$(-\infty, -2)$ and $(4, \infty)$

$f''(x) < 0$, f is $\frown$

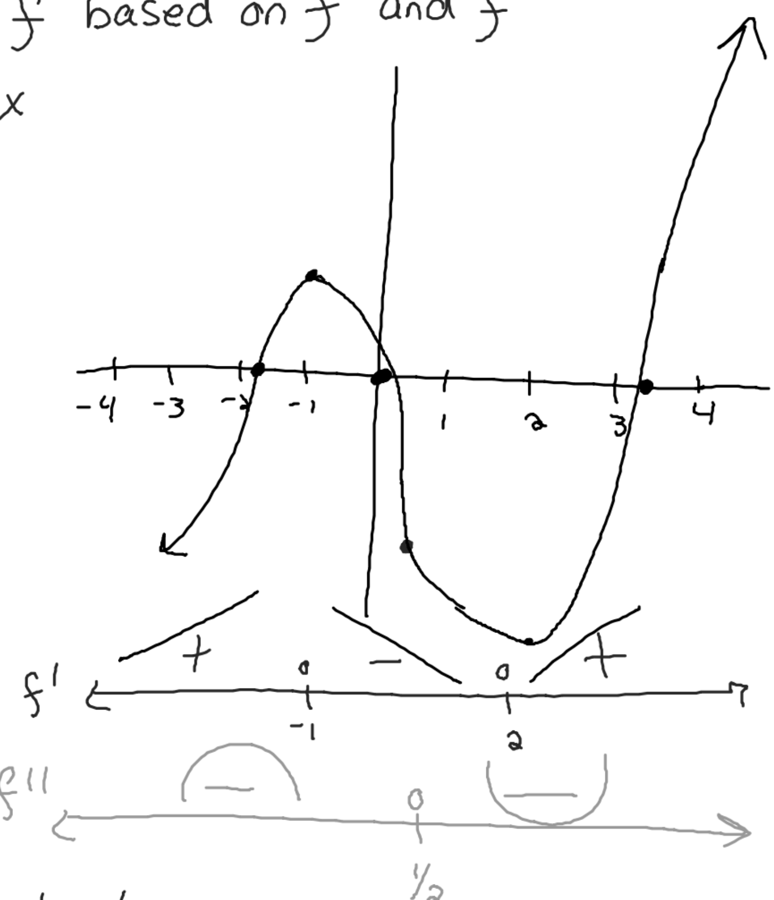
$(-2, 4)$

More practice with graphs:
need to sketch f based on f' and f''

a) $f(x) = 2x^3 - 3x^2 - 12x$

$f'(x) = 6x^2 - 6x - 12$

$f''(x) = 12x - 6$



$(0,0), (-1.8,0), (3.31,0)$

$f(x)$ x-int: ($y=0$) $0 = 2x^3 - 3x^2 - 12x$

$0 = \cancel{x}(2x^2 - 3x - 12)$

y-int: ($x=0$) $(0,0)$

$f'(x)$ inc: $(-\infty,-1)$ and $(2,\infty)$

dec: $(-1,2)$

max: $f(-1) =$ $(-1,7)$

min: $f(2) =$ $(2,-20)$

$f''(x)$ inflection pts

CUP $(1/2,\infty)$

CD $(-\infty, 1/2)$

I.P. at $x = \frac{1}{2}$

$(1/2, -13/2)$

75

#6 (fixed)

$$f(x) = (x^2+1)^{-1}$$

$$f'(x) = -2x(x^2-1)^{-2}$$

$$f''(x) = -2x\left[-2(x^2-1)^{-3}(2x)\right] + (x^2-1)^{-2}(-2)$$

$$= 8x^2(x^2-1)^{-3} - 2(x^2-1)^{-2}$$

$$= \frac{8x^2}{(x^2-1)^3} - \frac{2}{(x^2-1)^2} \cdot \frac{(x^2+1)}{(x^2+1)}$$

$$= \frac{8x^2 - 2x^2 - 2}{(x^2-1)^3} = \frac{6x^2-2}{(x^2-1)^3} = 0$$

$$6x^2 - 2 = 0$$

$$6x^2 = 2$$

$$x^2 = \frac{2}{6}$$

$$x = \pm\sqrt{\frac{1}{3}}$$

Ex 2

$$f(x) = \frac{x^2-4}{x^2-1}$$

$$f'(x) = \frac{6x}{(x^2-1)^2}$$

$$f''(x) = \frac{-6(3x^2+1)}{(x^2-1)^3}$$

HW TT p. 239
2, 3, 4, 6

x-int: $0 = \frac{x^2-4}{x^2-1}$ $\quad x^2-4=0$
$x^2=4$
$x=\pm 2$

y-int: $f(0)=\frac{0^2-4}{0^2-1}=4$

asymptotes: $x^2-1=0$ $\quad x^2=1$ $\quad x=\pm 1$ vertical
$y=1$ horizontal

max/min
$f'(x)=0$ $\quad \frac{6x}{(x^2-1)^2}=0$ $\quad 6x=0$
$x=0$

min at $x=0$ $\quad f(0)=4$ $\quad (0,4)$

I.P. $f''(x)=0$ $\quad -6(3x^2+1)=0$
$\cancel{-6}=0$ $\quad 3x^2+1=0$
$3x^2=-1$
no real solution $\Rightarrow$ no I.P.