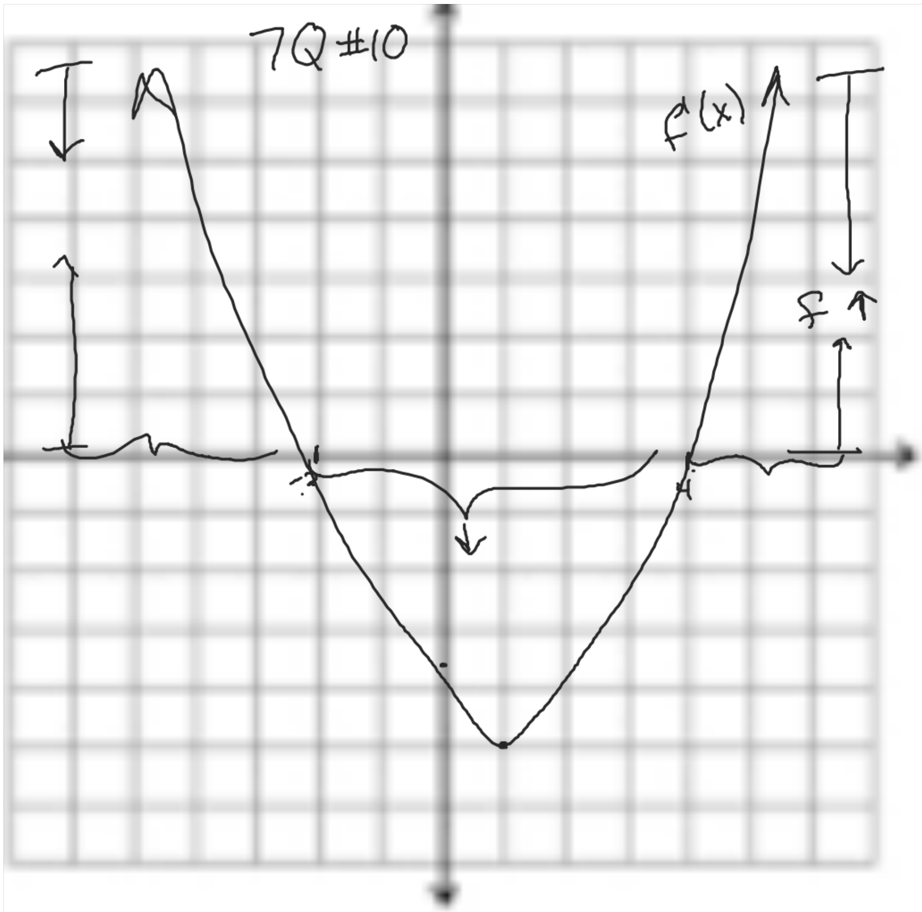



f is increasing

if $f'(x) > 0$

$(-\infty, -2)$

and

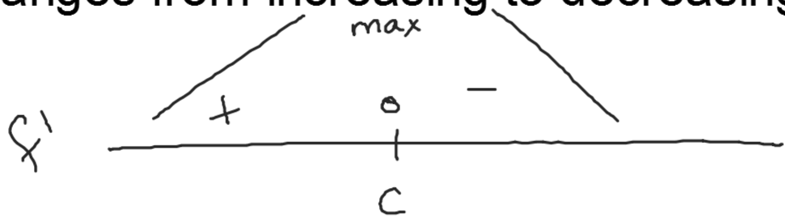
$(4, \infty)$

f is decreasing

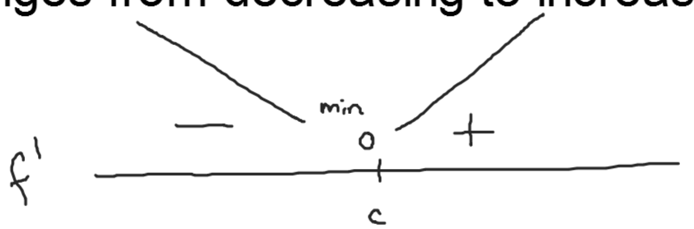
if $f'(x) < 0$

$(-2, 4)$

A function has a <u>relative maximum</u> (local max) when the function changes from increasing to decreasing.




A function has a <u>relative minimum</u> (local min) when the function changes from decreasing to increasing.




Relative maximums and minimums are called <u>Relative Extrema</u>

First Derivative Test is used to locate relative extrema of $f$.

If $f$ is defined at a critical number c, then:

1. If $f'(x)$ changes from positive to negative at $x=c$, then $f$ has a relative max at $(c, f(c))$.

2. If $f'(x)$ changes from negative to positive at $x=c$, then $f$ has a relative min at $(c, f(c))$.

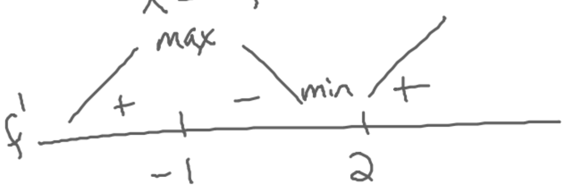
Ex a) $f(x) = 2x^3 - 3x^2 - 12x$

$f'(x) = 6x^2 - 6x - 12$

$6x^2 - 6x - 12 = 0$

factored

$x = 2, \quad x = -1$




Plug the critical numbers into the original function to find the points at which the extrema exist

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) = -2$

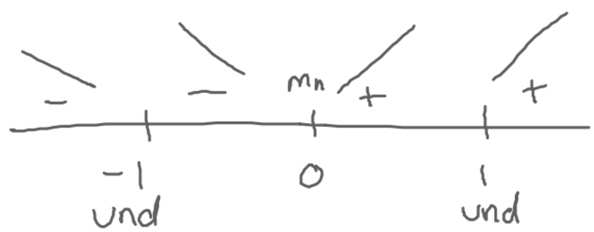
min is at $(2, -2)$

$f(1) = 2(1)^3 - 3(1)^2 - 12(1) = 7$

max at $(-1, 7)$

b) $f(x) = \dfrac{x^2 - 4}{x^2 - 1}$

$f'(x) = \dfrac{6x}{(x^2 - 1)^2}$




local min at

$f(0) = \dfrac{0^2 - 4}{0^2 - 1} = \dfrac{-4}{-1} = 4$

point is $(0, 4)$

HW7R p.234 #2-8 even

The Second derivative test:

if $f''(x) > 0$ for all $x$ in $(a,b)$, then $f$ is <u>convave up</u> on $(a,b)$

if $f''(x) < 0$ for all $x$ in $(a,b)$, then $f$ is <u>concave down</u> on $(a,b)$

The points on a graph where concavity changes are called <u>inflexion points.</u>

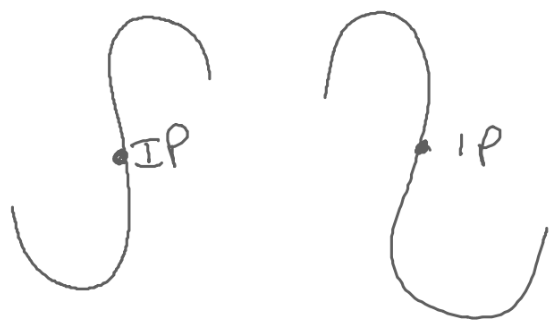
A point on the graph of $f$ is an inflexion point if $f''(x) = 0$ <u>and</u> $f''(x)$ changes sign.

Concave up (Cup)


tangent line ↓ curve

Concave down


tangent line is ↑



Ex) Use the 2nd derivative test to find the intervals where the function is CUP and Cd. Find the inflexion points.

a) $f(x) = 2x^3 - 3x^2 - 12x$

$f'(x) = 6x^2 - 6x - 12$

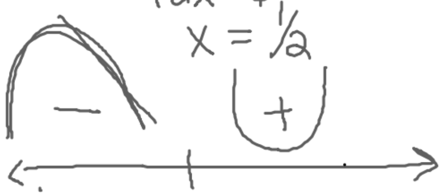
$f''(x) = 12x - 6$

To find I.P., set $f''(x) = 0$ and solve

$12x - 6 = 0$

$12x = 6$

$x = \frac{1}{2}$




$\frac{1}{2}$

plug into $f''$

$f''(0) = -6$

$f''(1) = 6$

concave up: $(\frac{1}{2}, \infty)$

concave down: $(-\infty, \frac{1}{2})$

$f(\frac{1}{2}) = \frac{-13}{2}$ $\quad (\frac{1}{2}, -\frac{13}{2})$ I.P.

b) $f(x) = \dfrac{x^2-4}{x^2-1}$

$f'(x) = \dfrac{6x}{(x^2-1)^2}$

$$f''(x) = \frac{(x^2-1)^{\cancel{2}}(6) - (6x)(2\cancel{(x^2-1)}(2x))}{(x^2-1)^{\cancel{4}\,3}}$$

$$f''(x) = \frac{-6(3x^2+1)}{(x^2-1)^3}$$

no I.P.

$\dfrac{-6(3x^2+1)}{\cancel{(x^2-1)^3}} = 0$ $\quad 3x^2 + 1 = 0$ $\quad x = \sqrt{-\frac{1}{3}}$ not real


$-1$ $\quad$ $1$

$f''$ is und $\quad$ und

Hw 7S p. 236 4-8 even