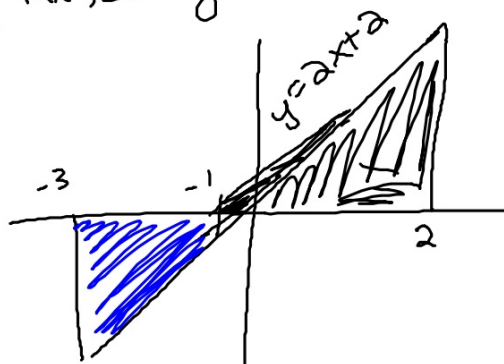


Definite Integrals, continued....

when  $f$  is non-negative  $a \leq x \leq b$ ,

$\int_a^b f(x) dx$  gives the area under the curve from  $x=a$  to  $x=b$



$$\int_{-1}^2 (2x+2) dx = 9$$

$$\int_{-3}^{-1} (2x+2) dx = -4$$

$$\begin{aligned} \int_{-3}^2 (2x+2) dx &= \int_{-3}^{-1} (2x+2) dx + \int_{-1}^2 (2x+2) dx \\ &= -4 + 9 = 5 \end{aligned}$$

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

### Properties of Definite Integrals

1.  $\int_a^b k f(x) dx = k \int_a^b f(x) dx$

2.  $\int_a^b (f(x) \pm g(x)) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$

3.  $\int_a^a f(x) dx = 0$

4.  $\int_a^b f(x) dx = - \int_b^a f(x) dx$

5.

Ex  $\int_0^2 f(x)dx=4$ ,  $\int_2^5 f(x)dx=12$ ,  $\int_0^2 g(x)dx=-3$ ,  $\int_0^4 g(x)dx=6$

a)  $\int_0^2 (3f(x) - g(x))dx = 3(4) - (-3) = 15$

b)  $\int_2^2 g(x)dx + \int_5^2 f(x)dx = 0 - 12 = -12$

c)  $\int_2^4 g(x)dx = \int_0^4 g(x)dx - \int_0^2 g(x)dx = 6 - (-3) = 9$

d)  $\int_{-3}^{-1} \frac{1}{2} f(x+3)dx = \frac{1}{2} \int_{-3}^{-1} f(x+3)dx$   
 $= \frac{1}{2} \int_0^2 f(x)dx = \frac{1}{2}(4) = 2$   
left 3

Hw 9H p. 308 #2-10 even, 11, 12

## 9.4 Fundamental Theorem of Calculus (FTC part 2)

$$\int_a^b f(x) dx = \left[ F(x) \right]_a^b = F(b) - F(a)$$

Shmoop  
integral

↑  
evaluated on  
the interval  
 $a \leq x \leq b$

↑  
Integral  
value  
at  
 $x=b$

↑  
Integral  
value  
at  
 $x=a$

use a GDC to evaluate

$$\int_0^2 (x^2+1) dx \approx 4.67$$

use FTC to evaluate

$$\begin{aligned} \int_0^2 (x^2+1) dx &= \left. \frac{1}{3}x^3 + x \right|_0^2 = \left( \frac{1}{3}(2)^3 + (2) \right) - \left( \frac{1}{3}(0)^3 + (0) \right) \\ &= \frac{8}{3} + 2 \approx 4.67 \end{aligned}$$

Ex) Evaluate without GDC

$$\begin{aligned} \text{a) } \int_{-2}^1 (u-1) du &= \left[ \frac{1}{2}u^2 - u \right]_{-2}^1 \\ &= \left[ \frac{1}{2}(1)^2 - 1 \right] - \left[ \frac{1}{2}(-2)^2 - (-2) \right] \\ &= -\frac{1}{2} - 4 = -\frac{9}{2} \end{aligned}$$

$$\text{b) } \int_2^3 \frac{1}{t} dt = \left[ \ln t \right]_2^3 = \ln(3) - \ln(2) = \ln\left(\frac{3}{2}\right)$$

$$\text{c) } \int_1^3 4x^2(x-1) dx = \int_1^3 (4x^3 - 4x^2) dx$$

$$= 4 \left[ \int_1^3 x^3 dx - \int_1^3 x^2 dx \right]$$

$$= 4 \left[ \frac{1}{4}x^4 - \frac{1}{3}x^3 \right]_1^3 = \left[ x^4 - \frac{4}{3}x^3 \right]_1^3$$

$$= \left[ 3^4 - \frac{4}{3}(3)^3 \right] - \left[ (1)^4 - \frac{4}{3}(1)^3 \right] = \frac{136}{3}$$

~~$u = x^2$   
 $du = 2x dx$~~

FTC with more complicated Integrals:

$$\begin{aligned}\textcircled{1} \int_1^5 \left( e^{ax} + \frac{1}{x^2} \right) dx &= \int_1^5 (e^{ax} + x^{-2}) dx \\ &= \left[ \frac{1}{a} e^{ax} - x^{-1} \right]_1^5 = \left( \frac{1}{a} e^{a(5)} - \frac{1}{5} \right) - \left( \frac{1}{a} e^{a(1)} - \frac{1}{1} \right) \\ &= \frac{1}{a} e^{10} - \frac{1}{5} - \frac{1}{a} e^a + 1 \\ &= \frac{1}{a} e^{10} - \frac{1}{a} e^a + \frac{4}{5}\end{aligned}$$

$$\begin{aligned} \textcircled{2} \int_0^3 \sqrt{3x+16} \, dx &= \int_0^3 (3x+16)^{1/2} \, dx \\ &= \left[ \frac{1}{3} \left[ \frac{2}{3} (3x+16)^{3/2} \right] \right]_0^3 = \left[ \frac{2}{9} (3x+16)^{3/2} \right] \\ &= \frac{2}{9} (3(3)+16)^{3/2} - \frac{2}{9} (3(0)+16)^{3/2} \\ &= \frac{2}{9} (25)^{3/2} - \frac{2}{9} (16)^{3/2} \\ &= \frac{2}{9} \left( (25)^{3/2} - (16)^{3/2} \right) \end{aligned}$$

$$\textcircled{3} \int_0^1 (2x^2+1)^3 (4x) dx$$

$$u = 2x^2 + 1$$
$$du = 4x dx$$

$$\int_0^1 u^3 du = \left[ \frac{1}{4} u^4 \right]_0^1$$

$$= \frac{1}{4} (1) - \frac{1}{4} (0)^4$$

=  
Mrs. Ruff will undo her  
damage on Monday

HW 9I p. 310 #1-6, 10  
HW 9J 1-4



Ex When you have a  $u$ -substitution, if you back-substitute the function, use the original's boundaries of integration. If you leave it in terms of  $u$ , you must change the limits of integration.



$$\int_0^1 (2x^2+1)^3 \underbrace{(4x)}_{du} dx$$

$$u = 2x^2 + 1$$

$$du = 4x dx$$

When  $x = 0$

$$u = 2(0)^2 + 1 = 1$$

When  $x = 1$

$$u = 2(1)^2 + 1 = 3$$

$$\int_1^3 u^3 du = \left[ \frac{1}{4} u^4 \right]_1^3$$

$$= \frac{1}{4} (3)^4 - \frac{1}{4} (1)^4$$

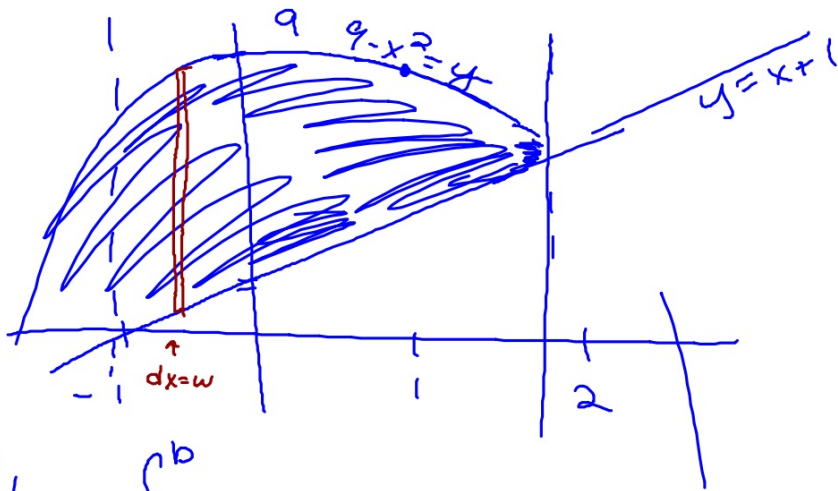
$$= \frac{81}{4} - \frac{1}{4} = \frac{80}{4} = 20$$

HW 9J p. 312

# 6, 8, 10

## 9.5 Area between two curves

Consider the functions  $f_1(x) = x+1$ ,  $f_2(x) = 9-x^2$   
between the lines  $x=-1$  and  $x=2$



add all  
vertical  
rectangles

$$\int_a^b (\text{top} - \text{bottom}) dx$$

$$\int_{-1}^2 [(9-x^2) - (x+1)] dx = \int_{-1}^2 (8-x^2-x) dx$$

$$8(2) - \frac{1}{3}(2)^3 - \frac{1}{2}(2)^2 - [8(-1) - \frac{1}{3}(-1)^3 - \frac{1}{2}(-1)^2]$$

$$(16 - \frac{8}{3} - 2) - (-8 + \frac{1}{3} - \frac{1}{2})$$

$$14 - \frac{8}{3} + 8 - \frac{1}{3} + \frac{1}{2}$$

$$22 - \frac{9}{3} + \frac{1}{2}$$

$$22 - 3 + \frac{1}{2}$$

$$19 + \frac{1}{2}$$

$$19\frac{1}{2}$$