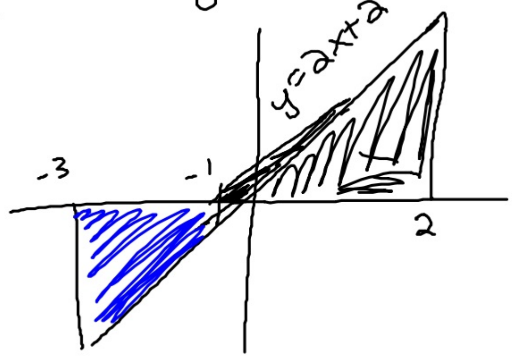
Definite Integrals, continued....

when f is non-negative $a \le x \le b$,

$\int_a^b f(x)\,dx$ gives the area under the curve from $x=a$ to $x=b$



$$\int_{-1}^{2} (2x+2)\,dx = 9$$

$$\int_{-3}^{-1} (2x+2)\,dx = -4$$

$$\int_{-3}^{2} (2x+2)\,dx = \int_{-3}^{-1} (2x+2)\,dx + \int_{-1}^{2} (2x+2)\,dx$$

$$= -4 + 9 = 5$$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

Properties of Definite Integrals

1. $\int_a^b K f(x)dx = K \int_a^b f(x)dx$

2. $\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

3. $\int_a^a f(x)dx = 0$

4. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

5.

Ex $\int_0^2 f(x)dx = 4$, $\int_2^5 f(x)dx = 12$, $\int_0^2 g(x)dx = -3$, $\int_0^4 g(x)dx = 6$

a) $\int_0^2 (3f(x) - g(x))dx = 3(4) - (-3) = 15$

b) $\int_2^2 g(x)dx + \int_5^2 f(x)dx = 0 - 12 = -12$

c) $\int_2^4 g(x)dx = \int_0^4 g(x)dx - \int_0^2 g(x)dx = 6 - (-3)$
$= 9$

d) $\int_{-3}^{-1} \frac{1}{2} f(\underset{\text{left } 3}{x+3})dx = \frac{1}{2}\int_{-3}^{-1} f(x+3)dx$

$= \frac{1}{2}\int_0^2 f(x)dx = \frac{1}{2}(4) = 2$

HW 9H p. 308 #2-10 even, 11, 12

9.4 Fundamental Theorem of Calculus (FTC part 2)

$$\int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b = F(b) - F(a)$$

shmoo integral

evaluated on the interval $a \le x \le b$

Integral value at $x=b$

Integral value at $x=a$

use a GDC to evaluate

$$\int_0^2 (x^2+1)\,dx \approx 4.67$$

use FTC to evaluate

$$\int_0^2 (x^2+1)\,dx = \frac{1}{3}x^3 + x\Big|_0^2 = \left(\frac{1}{3}(2)^3 + (2)\right) - \left(\frac{1}{3}(0)^3 + (0)\right)$$

$$= \frac{8}{3} + 2 \approx 4.67$$

Ex) Evaluate without GDC

a) $$\int_{-2}^{1} (u-1)\,du = \left[\frac{1}{2}u^2 - u\right]_{-2}^{1}$$

$$= \left[\frac{1}{2}(1)^2 - 1\right] - \left[\underbrace{\frac{1}{2}(-2)^2}_{2} - \underbrace{(-2)}_{+2}\right]$$

$$= -\frac{1}{2} - 4 = -\frac{9}{2}$$

b) $$\int_{2}^{3} \frac{1}{t}\,dt = \left[\ln t\right]_{2}^{3} = \ln(3) - \ln(2) = \ln\left(\frac{3}{2}\right)$$

c) $$\int_{1}^{3} 4x^2(x-1)\,dx = \int_{1}^{3} (4x^3 - 4x^2)\,dx$$

$\cancel{u = x^2}$
$\cancel{du = 2x\,dx}$

$$= 4\left[\int_{1}^{3} x^3\,dx - x^2\,dx\right]$$

$$= 4\left[\frac{1}{4}x^4 - \frac{1}{3}x^3\right]_{1}^{3} = \left[x^4 - \frac{4}{3}x^3\right]_{1}^{3}$$

$$= \left[3^4 - \frac{4}{3}(3)^3\right] - \left[(1)^4 - \frac{4}{3}(1)^3\right] = \frac{136}{3}$$

FTC with more complicated Integrals:

(1) $\int_1^5 \left(e^{2x} + \frac{1}{x^2} \right) dx = \int_1^5 (e^{2x} + x^{-2})\, dx$

$= \left[\frac{1}{2}e^{2x} - x^{-1} \right]_1^5 = \left(\frac{1}{2}e^{2(5)} - \frac{1}{5} \right) - \left(\frac{1}{2}e^{2(1)} - \frac{1}{1} \right)$

$= \frac{1}{2}e^{10} - \frac{1}{5} - \frac{1}{2}e^{2} + 1$

$= \frac{1}{2}e^{10} - \frac{1}{2}e^{2} + \frac{4}{5}$

② $\int_0^3 \sqrt{3x+16}\,dx = \int_0^3 (3x+16)^{1/2}\,dx$

$$= \left[\frac{1}{3}\left[\frac{2}{3}(3x+16)^{3/2}\right]\right]_0^3 = \left[\frac{2}{9}(3x+16)^{3/2}\right]$$

$$= \frac{2}{9}(3(3)+16)^{3/2} - \frac{2}{9}(3(0)+16)^{3/2}$$

$$= \frac{2}{9}(25)^{3/2} - \frac{2}{9}(16)^{3/2}$$

$$= \frac{2}{9}\left((25)^{3/2} - (16)^{3/2}\right)$$

3 $\int_0^1 \underbrace{(2x^2+1)}_{u}{}^3 \underbrace{(4x)\,dx}_{du}$

$u = 2x^2+1$

$du = 4x\,dx$

$\int_0^1 u^3\,du = \left[\frac{1}{4}u^4\right]_0^1$

$= \frac{1}{4}(\quad) - \frac{1}{4}(0)^4$

$=$

Hw 9I p. 310 #1-6, 10

Hw 9J 1-4

Mrs. Ruff will undo her damage on Monday

Ex) when you have a u-substitution, if you back-substitute the function, use the originals' boundaries of integration. If you leave it in terms of u, you must change the limits of integration.

$$\int_0^1 (\underbrace{2x^2+1}_{u})^3 \underbrace{(4x)\,dx}_{du}$$

$u = 2x^2 + 1$

$du = 4x\,dx$

when $x = 0$

$u = 2(0)^2 + 1 = 1$

when $x = 1$

$u = 2(1)^2 + 1 = 3$

$$\int_1^3 u^3\,du = \left[\frac{1}{4}u^4\right]_1^3$$

$$= \frac{1}{4}(3)^4 - \frac{1}{4}(1)^4$$

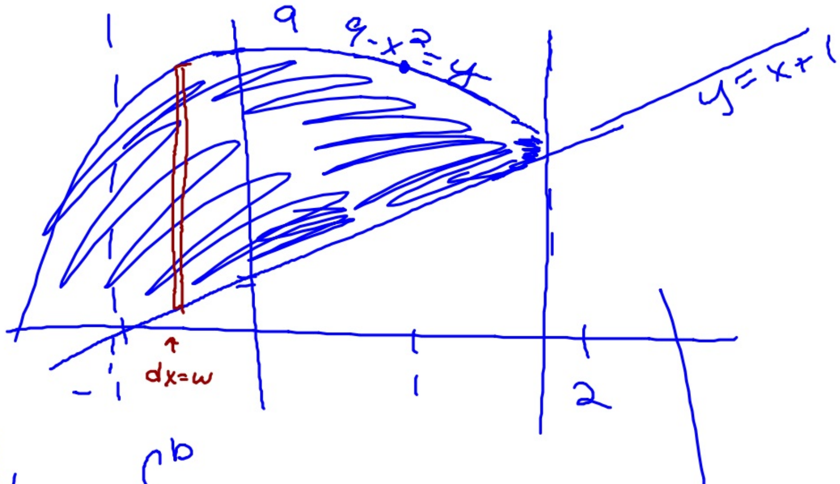
$$= \frac{81}{4} - \frac{1}{4} = \frac{80}{4} = 20$$

HW 9J P. 312

6, 8, 10

9.5 Area between two curves

Consider the functions $f_1(x) = x+1$, $f_2(x) = 9-x^2$
between the lines $x=-1$ and $x=2$



add all vertical rectangles

$$\int_a^b (\text{top}-\text{bottom})\,dx$$

$$\int_{-1}^{2}\left[(9-x^2)-(x+1)\right]dx = \int_{-1}^{2}(8-x^2-x)\,dx$$

$$8(2)-\frac{1}{3}(2)^3-\frac{1}{2}(2)^2-\left[8(-1)-\frac{1}{3}(-1)^3-\frac{1}{2}(-1)^2\right]$$

$$\left(16-\frac{8}{3}-2\right)-\left(-8+\frac{1}{3}-\frac{1}{2}\right)$$

$$14-\frac{8}{3}+8-\frac{1}{3}+\frac{1}{2}$$

$$22-\frac{9}{3}+\frac{1}{2}$$

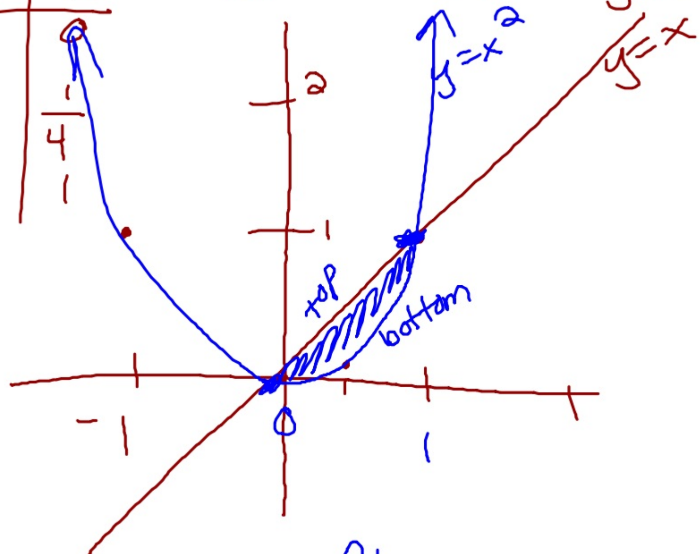
$$22-3+\frac{1}{2}$$

$$19+\frac{1}{2}$$

$$19\frac{1}{2}$$

Ex] Graph the area between $y=x$ and $y=x^2$ write down an expression that gives the area of the region and find the region.

x	x^2
0	0
$\frac{1}{2}$	$\frac{1}{4}$
1	1



need to find the points of intersection:

$$x = x^2$$

$$x^2 - x = 0$$

$$x(x-1) = 0$$

$x=0$ and $x=1$
a b

$$\int_0^1 (x - x^2)\,dx = \frac{1}{2}x^2 - \frac{1}{3}x^3 \Big|_0^1 = \frac{1}{6}$$

Ex] consider $f(x) = x^4 - x^2$

#5 9K

HW 9K
p. 316
#1-4, 5d, 6-10

a) find the x intercepts

$x^4 - x^2 = 0$

$x^2(x^2-1) = 0$

$x = 0 \quad x^2 = 1$

$x = \pm 1$

b) Find the coordinates of the min + max

$f'(x) = 4x^3 - 2x = 0$ $\quad 2x = 0$, $x = 0$

$2x(2x^2-1) = 0$ $\quad x^2 = \frac{1}{2}$

Coordinates: $(0,0)$ $\quad x = \pm\frac{1}{\sqrt{2}}$

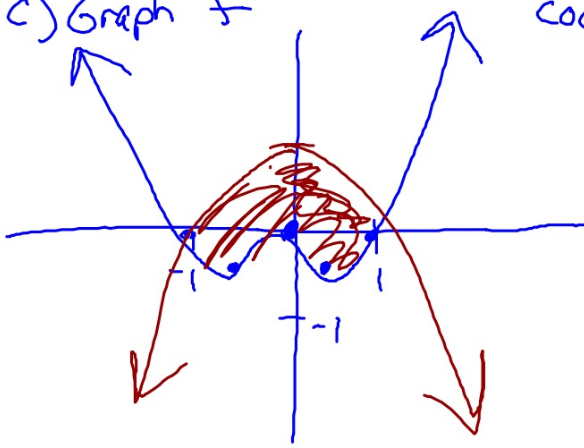
$\left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$ or $\frac{\pm\sqrt{2}}{2}$

$\left(\frac{1}{\sqrt{2}}, -\frac{1}{4}\right)$

$\left(-\frac{1}{\sqrt{2}}\right)^4 - \left(-\frac{1}{\sqrt{2}}\right)^2 =$

c) Graph f

Sketch $g(x) = 1 - x^2$



$\int_{-1}^{1}$