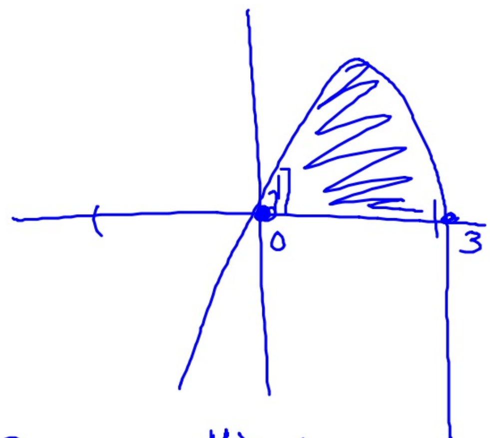
#3 $f(x) = 3x - x^2$ and the x-axis

$$3x - x^2 = 0$$

$$x(3-x) = 0$$

$$\underset{a}{x=0} \text{ or } \underset{b}{x=3}$$



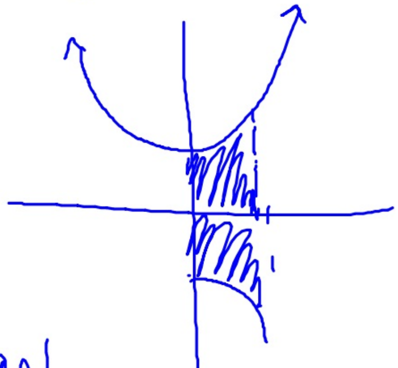
$$\int_0^3 \pi (3x-x^2)^2\,dx = \pi\int_0^3 (9x^2 - 6x + x^4)\,dx$$

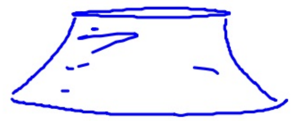
$$(3x-x^2)(3x-x^2)$$

$$= \pi\left[3x^3 - 3x^2 + \frac{1}{5}x^5\right]_0^3 = \pi\left(3(3)^3 - 3(3)^2 + \frac{3^5}{5}\right) - \pi(0)$$

$$= \frac{81\pi}{10}$$

Ex) FIND THE VOLUME OF A SOLID FORMED BY THE REGION UNDER THE CURVE $y = x^2+1$ AND THE X-AXIS BETWEEN $x=0$ AND $x=1$





$$\int_0^1 \pi (x^2+1)^2\,dx = \pi \int_0^1 (x^4+2x^2+1)\,dx$$

$$= \pi \left[\frac{1}{5}x^5 + \frac{2}{3}x^3 + x\right]_0^1$$

$$= \pi\left[\frac{1}{5}(1)^5 + \frac{2}{3}(1)^3 + 1\right] - \pi\left[0\right]$$

$$= \frac{28\pi}{15}$$

HW 9N
p. 320
#1, 3, 5, 6

9.7 DEFINITE INTEGRALS WITH LINEAR MOTION AND OTHER PROBLEMS

- find the change in a function over time

Recall:

$$v(t) = s'(t) \implies s(t) = \int v(t)\,dt$$

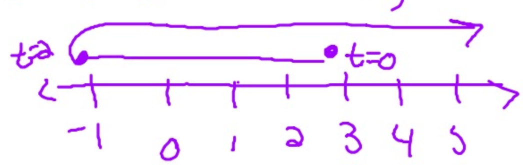
$$a(t) = s''(t) = v'(t) \implies v(t) = \int a(t)\,dt$$

$$\boxed{\int_{t_1}^{t_2} v(t)\,dt = s(t_2) - s(t_1) \text{ is the change in displacement from } t_1 \text{ to } t_2}$$

t in sec, s in m.

Ex) if $s(t) = t^2 - 4t + 3,\ t \geq 0$ $\quad s(0) = 3,\ s(2) = -1$

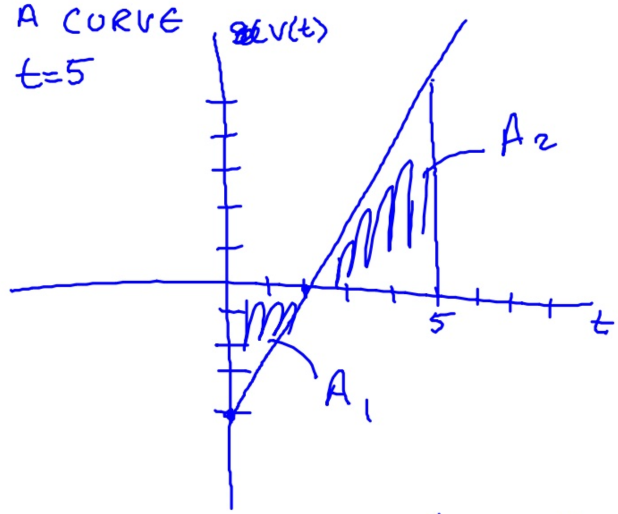
motion diagram



displacement is -4

RELATE THIS TO THE AREA UNDER A CURVE

$v(t) = 2t - 4$ from $t=0$ to $t=5$



$$\int_0^5 v(t)\,dt = -A_1 + A_2$$
$$= A_2 - A_1$$

$$\int_0^5 (2t-4)\,dt = \left[t^2 - 4t\right]_0^5$$
$$= (5^2 - 4(5)) - (0)$$
$$= 25 - 20 = 5$$

$$-\tfrac{1}{2}(2)(4) + \tfrac{1}{2}(3)(6)$$
$$= 4 \qquad = 9$$

displacement in 5 seconds is 5 meters

the particle has moved 5 metres right from its original spot

Total distance traveled in 5 seconds

Add A_1 and A_2 = ~~4+5~~ $4+9=13$ metres