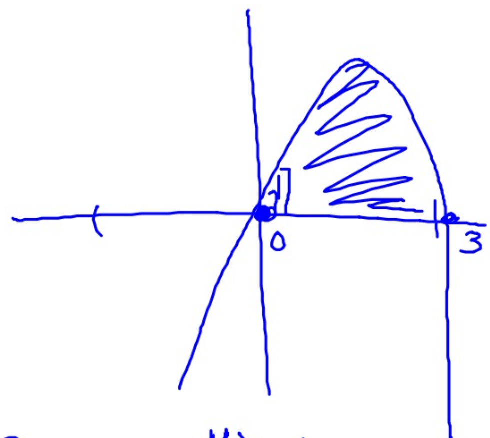
#3 $f(x) = 3x - x^2$ and the x-axis

$$3x - x^2 = 0$$

$$x(3 - x) = 0$$

$$\underset{a}{x = 0} \text{ or } \underset{b}{x = 3}$$

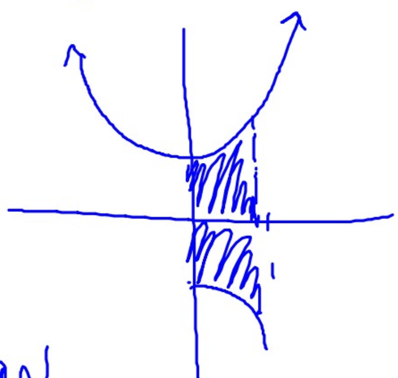


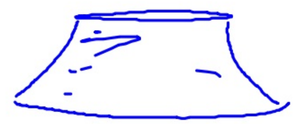
$$\int_0^3 \pi \underbrace{(3x - x^2)^2}_{(3x-x^2)(3x-x^2)} dx = \pi \int_0^3 (9x^2 - 6x + x^4)\, dx$$

$$= \pi \left[3x^3 - 3x^2 + \frac{1}{5}x^5 \right]_0^3 = \pi\left(3(3)^3 - 3(3)^2 + \frac{3^5}{5}\right) - \pi(0)$$

$$= \frac{81\pi}{10}$$

Ex) FIND THE VOLUME OF A SOLID FORMED BY THE REGION UNDER THE CURVE $y = x^2 + 1$ AND THE X-AXIS BETWEEN $x = 0$ AND $x = 1$





$$\int_0^1 \pi (x^2+1)^2\,dx = \pi \int_0^1 (x^4 + 2x^2 + 1)\,dx$$

$$= \pi \left[\frac{1}{5}x^5 + \frac{2}{3}x^3 + x \right]_0^1$$

$$= \pi \left[\frac{1}{5}(1)^5 + \frac{2}{3}(1)^3 + 1 \right] - \pi \left[0 \right]$$

$$= \frac{28\pi}{15}$$

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9.7 DEFINITE INTEGRALS WITH LINEAR MOTION AND OTHER PROBLEMS

- find the change in a function over time

Recall:

$$v(t) = s'(t) \implies s(t) = \int v(t)\,dt$$

$$a(t) = s''(t) = v'(t) \implies v(t) = \int a(t)\,dt$$

$$\boxed{\int_{t_1}^{t_2} v(t)\,dt = s(t_2) - s(t_1) \text{ is the change in displacement from } t_1 \text{ to } t_2}$$

t in sec, s in m.

Ex) if $s(t) = t^2 - 4t + 3,\ t \geq 0$ $\quad s(0) = 3,\ s(2) = -1$

motion diagram

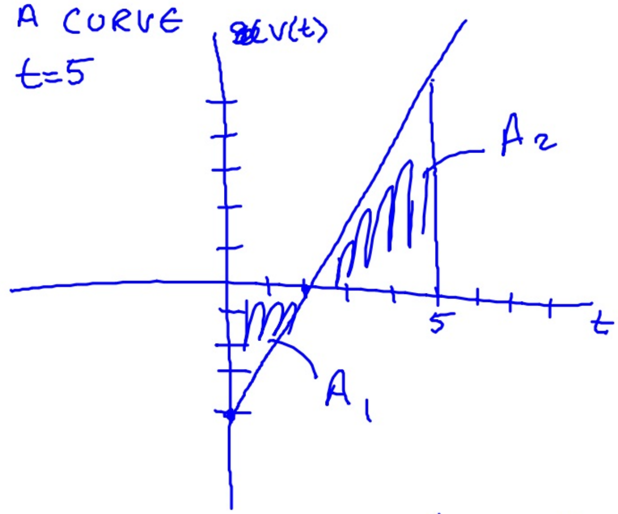
displacement is -4

RELATE THIS TO THE AREA UNDER A CURVE

$v(t) = 2t - 4$ from $t=0$ to $t=5$

$$\int_0^5 v(t)\,dt = -A_1 + A_2$$

$$= A_2 - A_1$$



$$-\tfrac{1}{2}(2)(4) + \tfrac{1}{2}(3)(6)$$

$= 4 \qquad = 5$

$$\int_0^5 (2t-4)\,dt = \left[t^2 - 4t\right]_0^5$$

$$= (5^2 - 4(5)) - (0)$$

$$= 25 - 20 = 5$$

displacement in 5 seconds is 5 meters

the particle has moved 5 metres right from its original spot

Total distance traveled in 5 seconds

Add A_1 and A_2 = ~~4+5~~ 4+9 = 13 metres

6) a) $\int_1^a \pi \left(\frac{1}{\sqrt{x}}\right)^2 dx = \int_1^a \pi \left(\frac{1}{x}\right) dx$

$= \left[\pi \ln(x)\right]_1^a = \pi \left[\ln a - \ln(1)\right]$

$= \pi \ln a$

b) $V = 3\pi$ find a

$3\cancel{\pi} = \cancel{\pi} \ln a$

$3 = \ln a$

$e^3 = \cancel{e}^{\cancel{\ln} a}$

$e^3 = a$

5 b) $\int_0^{\ln 4} \pi (e^{1/4 x})^2 dx = 2\pi$

$= \pi \int_0^{\ln 4} e^{\frac{1}{2}x} dx = \pi \left[2e^{\frac{1}{2}x}\right]_0^{\ln 4}$

$= \pi \left[2e^{\frac{1}{2}\ln 4} - 2e^{\frac{1}{2}(0)}\right]$

$= \pi \left[2e^{\ln 4^{1/2}} - 2e^{(0)}\right]$

$= \pi \left[2\cancel{e^{\ln}}2 - 2\right]$

$= \pi \left[2(2) - 2\right]$

$= 2\pi$

$\left(e^{\frac{1}{4}x}\right)^2 = e^{\frac{1}{4}\cdot 2x}$

$= e^{\frac{1}{2}x}$

b) $K\pi = 2\pi$

$K = 2$

Ex The displacement function for a particle moving along a horizontal line is given by:

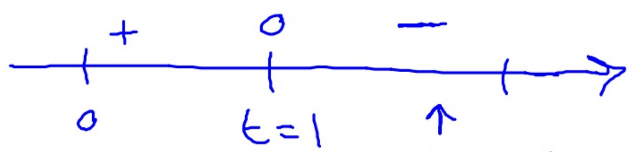
$s(t) = 8 + 2t - t^2$ for $t \geq 0$ t in sec
s in metres

a) Find the velocity of the particle at time t.

$$v(t) = s'(t) = 2 - 2t$$

b) Find when the particle is moving left or right.

$$2 - 2t = 0$$
$$-2t = -2$$
$$t = 1 \text{ sec}$$



choose $t=2$

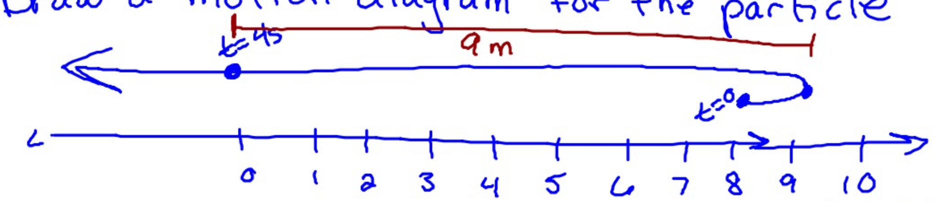
$v(2) = -2$

$\therefore$ moving $\leftarrow$

moves right $0 < t < 1$

moves left $t > 1$

c) Draw a motion diagram for the particle



$S(4) = 0\,m$

1m

when $t = 0$

$$S(0) = 8 + 2(0) - (0)^2 = 8\ m.$$

$t = 1$ $\quad S(1) = 8 + 2(1) - (1)^2 = 9\ m$

d) write definite integrals to find displacement and total distance traveled on $0 \le t \le 4$. Use GOC to calculate.

Displacement:

$$\int_0^4 v(t)\,dt = \int_0^4 (2 - 2t)\,dt = -8\ m$$

Distance:

$$\int_0^4 |v(t)|\,dt = 10\ m$$

EX) velocity function ~~~~ is shown.
find change in displacement and
total distance traveled

$A = \frac{1}{2}(b_1 + b_2) \cdot h$

change in displacement:

$$\int_0^{16} v(t)\,dt = -A_1 + A_2 - A_3$$

$$= -\frac{1}{2}(4)(4) + \frac{1}{2}(8)(4) - \frac{1}{2}(4+1)(2)$$