14.3 Integral of Sine & Cosine

Rules still Apply!

$$\int \sin(x)\,dx = -\cos(x) + C$$

since $\frac{d}{dx} -\cos(x) = -(-\sin x)$

$= \sin x$

$$\int \cos x\,dx = \sin x + C$$

$\frac{d}{dx} \sin x + C = \cos x$

Ex)

a. $\int 3\sin x\,dx = 3\int \sin x\,dx = -3\cos x \boxed{+C}$

b. $\int \cos(4x-6)\,dx = \frac{1}{4}\sin(4x-6) + C$

c. $\int \underbrace{e^x}_{du} \sin(\underbrace{e^x}_{u}) \underbrace{dx}_{du}$ $\quad u = e^x$, $du = e^x dx$

$\int \sin(u)\,du = -\cos(u) + C$

$= -\cos(e^x) + C$

d) $\int \underline{x^3} \cos(3x^4)\,\underline{dx}$ $\quad u = 3x^4$, $du = 12x^3 dx$

$\frac{1}{12}\int \cos(u)\,du$ $\quad \frac{1}{12}du = x^3 dx$

$= \frac{1}{12}(\sin(3x^4)) + C$

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FTC For Definite Integrals

$$\int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b = F(b) - F(a)$$

Ex) a. $\displaystyle\int_0^{\frac{\pi}{4}} 2\cos x\,dx = 2\int_0^{\frac{\pi}{4}} \cos x\,dx$

$$= 2\Big[\sin(x)\Big]_0^{\frac{\pi}{4}}$$

$$= 2\left(\sin\left(\tfrac{\pi}{4}\right) - \sin(0)\right)$$

$$= 2\left(\frac{\sqrt{2}}{2}\right) - 0 = \sqrt{2}$$

b) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin(2x)\cos^3(2x)\,dx$

$u = \cos(2x)$

$du = -\sin(2x)(2)$

$du = -2\sin(2x)$

$-\frac{1}{2}du = \sin(2x)$

* When you do a u-sub with a <u>definite integral</u> you must change the limits of integration!!

if $x = \frac{\pi}{4}$, then $u = \cos\left(2\left(\frac{\pi}{4}\right)\right)$
$= \cos\left(\frac{\pi}{2}\right)$
$= 0$ lower

if $x = \frac{\pi}{2}$, then $u = \cos\left(2\left(\frac{\pi}{2}\right)\right) = \cos(\pi) = -1$ upper

$$-\frac{1}{2}\int_0^{-1} u^3\,du = -\frac{1}{2}\left[\left(\frac{1}{4}\right)u^4\right]_0^{-1} = -\frac{1}{2}\left[\frac{(-1)^4}{4} - \frac{0^4}{4}\right]$$

$$= -\frac{1}{2}\left[\frac{1}{4}\right] = -\frac{1}{8}$$

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