4 $\int_{\ln\frac{\pi}{4}}^{\ln\frac{\pi}{3}} e^x \cos(e^x)dx \Rightarrow u = e^x; \frac{du}{dx} = e^x;$

when $x = \ln\frac{\pi}{4}$ then $u = \frac{\pi}{4}$ and

when $x = \ln\frac{\pi}{3}$ then $u = \frac{\pi}{3}$

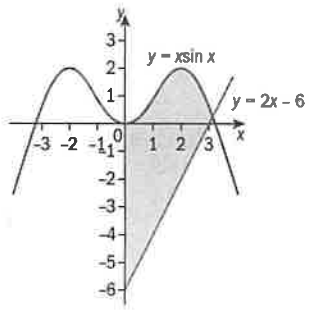
$$\int_{\ln\frac{\pi}{4}}^{\ln\frac{\pi}{3}} e^x \cos(e^x)dx = \int_{x=\ln\frac{\pi}{4}}^{x=\ln\frac{\pi}{3}} \left(\frac{du}{dx}\right)\cos(u)dx = \int_{u=\frac{\pi}{4}}^{u=\frac{\pi}{3}} \cos u du$$

$$= \left[\sin u\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \sin\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{3}-\sqrt{2}}{2}$$

$$\int_{\ln\frac{\pi}{4}}^{\ln\frac{\pi}{3}} e^x \cos(e^x)dx \approx 0.159 \text{ and } \frac{\sqrt{3}-\sqrt{2}}{2} \approx 0.159$$

Exercise 14G

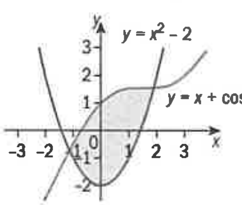
1 $y = x\sin x$ and $2x - 6$



$x\sin x = 2x - 6 \Rightarrow x = 3.1$

$\int_0^{3.041} (x\sin x - 2x + 6)dx \approx 12.1$

2 $y = x^2 - 2$ and $y = x + \cos x$



$x^2 - 2 = x + \cos x \Rightarrow x \approx -1.135, 1.891$

$\int_{-1.135}^{1.891} \left[(x + \cos x) - (x^2 - 2)\right] \approx 6.31$

3 $\int_0^k \cos x dx = \frac{1}{2}$ and $0 \le k \le \frac{\pi}{2}$

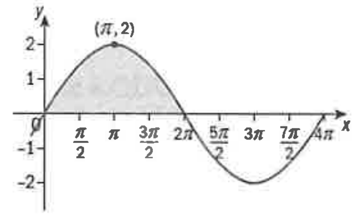
$\int_0^k \cos x dx = \frac{1}{2} \Rightarrow [\sin x]_0^k = \frac{1}{2}$

5 a $f(x) = a\sin(bx)$

The sine function has a vertical stretch by a factor of 2 $\Rightarrow a = 2$.

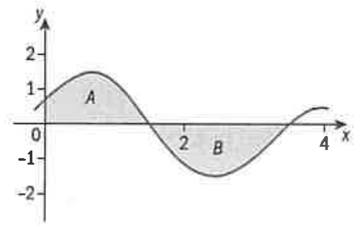
Since the period of f is 4π we have

$\frac{2\pi}{|b|} = 4\pi \Rightarrow b = \frac{1}{2}$



$\int_0^{2\pi} 2\sin\left(\frac{1}{2}x\right)dx = 8$

6 $y = \cos x + \sin 2x$



a i $y = \cos x + \sin 2x \Rightarrow y = \cos x + 2\sin x \cos x$

$\Rightarrow y = \cos x(1 + 2\sin x)$

$y = \cos x(c + d\sin x) \Rightarrow c = 1$ and $d = 2$

ii $\cos x(1 + 2\sin x) = 0 \Rightarrow \cos x = 0$ or $\sin x = -\frac{1}{2}$

$\Rightarrow x = \frac{\pi}{2}, \frac{7\pi}{6}$

b i $\int_0^{\frac{\pi}{2}} \left[\cos x + \sin(2x)\right]dx = 2$

ii $2 - \int_{\frac{\pi}{2}}^{\frac{7\pi}{6}} \left[\cos x + \sin(2x)\right]dx = 4.25$

c $\pi\int_0^{\frac{\pi}{2}} \left[\cos x + \sin(2x)\right]^2 dx \approx 9.12$

Exercise 14H

1 a $s(t) = e^t \sin t$