

Methods

14.4 Revising Linear Motion

$$\text{Displacement} = s(t)$$

$$\text{Velocity } v(t) = \frac{ds}{dt} = s'(t)$$

$$\text{Acceleration } a(t) = \frac{dv}{dt} = v'(t) = s''(t)$$

$$\text{total distance from } t_1 \text{ to } t_2 = \int_{t_1}^{t_2} |v(t)| dt$$

Ex A particle moves along a horizontal line. The particle's displacement, in metres, from origin O is $s(t) = 5 - 2\cos(3t)$ in t seconds

a) Find the particle's velocity and accel. at any time t.

$$v(t) = s'(t) = -2(-\sin(3t))(3) = 6\sin(3t)$$

$$a(t) = v'(t) = 6\cos(3t)(3) = 18\cos(3t)$$

b) Find the initial displacement, vel., accel

$$t=0$$

$$s(0) = 5 - 2\cos(3 \cdot 0) = 5 - 2\cos(0) = 5 - 2(1) = 3 \text{ m}$$

$$v(0) = 6\sin(3 \cdot 0) = 6\sin(0) = 6(0) = 0 \text{ ms}^{-1}$$

$$a(0) = 18\cos(0) = 18 \cdot 1 = 18 \text{ ms}^{-2}$$

c) Find when part is moving \rightarrow , \leftarrow , stopped during time $0 \leq t \leq \pi$

For stopped: $v(t) = 0$

$$\begin{aligned} & \sin(3t) = 0 \\ & \sin(3t) = 0 \quad 3t = 0, \pi, 2\pi, 3\pi \\ & t = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi \end{aligned}$$

$$t = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$$

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right when $v(t) > 0 \Rightarrow (0, \pi/3) \text{ and } (\frac{2\pi}{3}, \pi)$

left when $v(t) < 0 \Rightarrow (\pi/3, \frac{2\pi}{3})$

d) definite integral for total distance $0 \leq t \leq \pi$

$$\begin{aligned} \int_0^\pi |v(t)| dt &= \int_0^\pi |\sin(3t)| dt = 6 \int_0^{\pi/3} \sin(3t) dt \\ &= \left[\frac{6}{3} (-\cos(3t)) \right]_0^{\pi/3} = 12 \text{ m} \end{aligned}$$