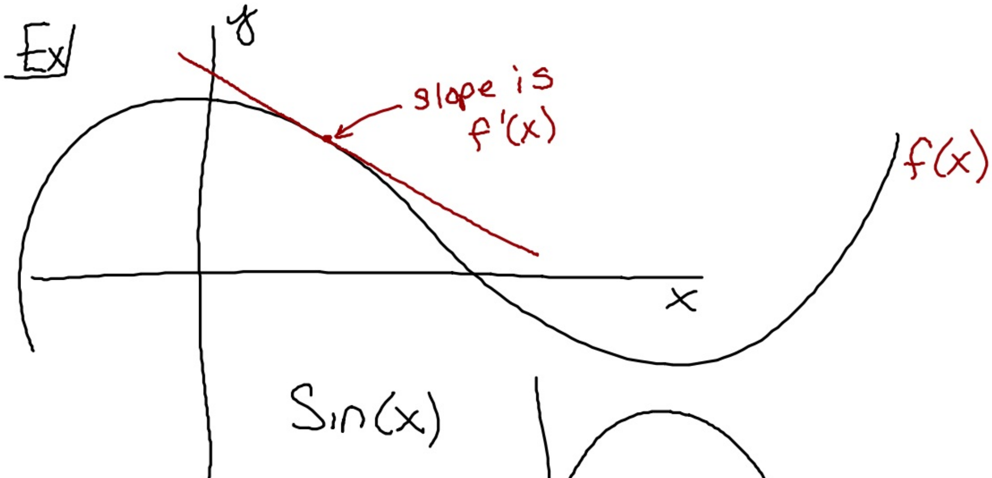
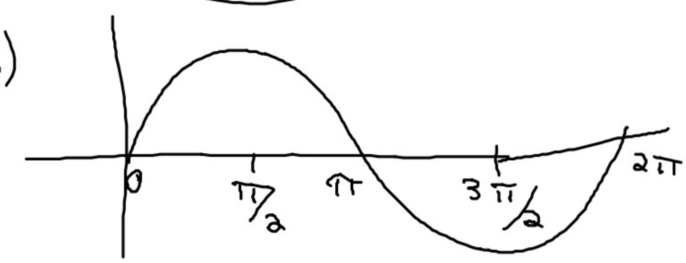
14.1 DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

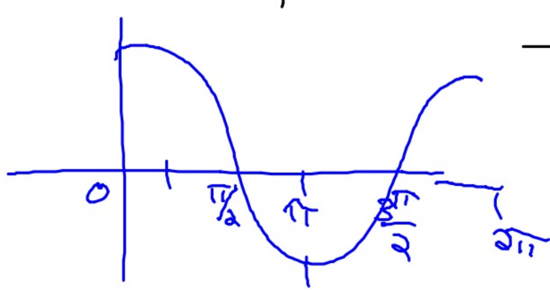
RECALL: A <u>derivative</u> is the instantaneous slope of a line tangent to the function at any given *x*.

Ex



Sin(x)





$f(x) = \sin x$

$f'(x) = \cos(x)$ thus, $\sin'(x) = \cos(x)$

$\cos(x + \frac{\pi}{2}) = -\sin x$ and $\cos'(x) = -\sin(x)$

to find $\tan'(x)$, use the quotient rule

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\tan'(x) = \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2(x)$$

$$= 1 + \tan^2(x)$$

$$\frac{d}{dx}\sin(x) = \cos(x) \implies \int \cos(x)\,dx = \sin(x) + C$$

$$\frac{d}{dx}\cos(x) = -\sin(x) \implies \int \sin(x)\,dx = -\cos(x) + C$$

$$\frac{d}{dx}\tan(x) = \sec^2(x) \implies \int \sec^2(x)\,dx = \tan(x) + C$$

$\boxed{Ex}$ FIND THE DERIVATIVES OF

a) $f(x) = \sin(x) + \cos(x)$

$f'(x) = \cos(x) - \sin(x)$

b) $y = \cos(t^2)$

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$$y' = -\sin(t^2)(2t)$$
$$= -2t\sin(t^2)$$

Power Rule

c) $y = \frac{1}{\tan(x)} = \tan^{-1}(x)$

$$\frac{d}{dx}(\tan(x))^{-1} = -1(\tan(x))^{-2}(\sec^2(x))$$

$$= \frac{-\sec^2(x)}{\tan^2(x)} = \frac{-1}{\tan^2(x)\cos^2(x)} =$$

$$= \frac{-1}{\frac{\sin^2(x)}{\cancel{\cos^2(x)}}(\cancel{\cos^2}(x))} = -\csc^2(x)$$

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d) $f(x) = \sin^3(2x)$

$$f'(x) = 3\sin^2(2x)(\cos(2x))(2)$$

Power rule → chain

$$= 6\sin^2(2x)\cos(2x)$$