[Maximum mark: 7]

$$(2x+1)^4 = \binom{4}{0}(2x)^4(1)^0 + \binom{4}{1}(2x)^3(1)^1 + \underbrace{\binom{4}{2}(2x)^2(1)^2} + \binom{4}{3}(2x)^1(1)^3 + \binom{4}{4}2x^0(1$$

In the expansion of $(2x+1)^n$, the coefficient of the term in x^2 is $40n$, where $n \in \mathbb{Z}^+$. Find n.

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n$$

$$\_\,x^n + \_\,x^{n-1} + \cdots + 40n\,x^2 + \_\_\,x + 1^n$$

$$\binom{n}{n-2}(a)^2(b)^{n-2} \quad \binom{n}{n-1}a^1b^{n-1} \quad \binom{n}{n}a^0b^n$$

$a = 2x$

$b = 1$

$$\binom{n}{n-2}(2x)^2(1)^{n-2} = 40n\,x^2$$

$$\binom{n}{2}(2x)^2(1)^{n-2} = 40n\,x^2$$

$$4x^2 = 40n\,x^2$$

$$\frac{n!}{2!(n-2)!}$$

$$\frac{n!}{2!(n-2)!}\, 4\cancel{x^2} = 40n\cancel{x^2}$$

$$\frac{n!}{2!(n-2)!} = 10n$$

$$\frac{n(n-1)\cancel{(n-2)!}}{2\cdot 1\,\cancel{(n-2)!}} = 10n$$

$$2\cdot\frac{n(n-1)}{2} = 10n\cdot 2$$

$$n(n-1) = 20n$$

$4! = 4\cdot 3\cdot 2\cdot 1$

$= 4(4-1)(4-2)(4-1)$

OPTION 1

$n^2 - n = 20n$

$n^2 - 21n = 0$

$n(n-21) = 0$

$n=0,\ n=21$

OPTION 2

$\frac{n}{n}(n-1) = 20\frac{n}{n}$

$n - 1 = 20$

$n = 21$

$n \in \mathbb{Z}^+$

$\Rightarrow n \neq 0 \therefore n = 21$

[2 marks]

The values in the fourth row of Pascal's triangle are shown in the following table.

1	4	6	4	1

Write down the values in the fifth row of Pascal's triangle.

1 5 10 10 5 1

x^5 x^4 x^3

1b. *[5 marks]*

Hence or otherwise, find the term in x^3 in the expansion of $(2x+3)^5$.

$n=5,\ r=\cancel{3}2,\ a=(2x),\ b=3$

$\binom{n}{r}(a)^{n-r}(b)^r \Rightarrow \binom{5}{2}(2x)^3(3)^2 = 10(8x^3)(9) = 720x^3$

$\frac{5!}{3!2!} = \frac{5\cdot4\cdot\cancel{3!}}{\cancel{3!}\cdot2\cdot1} = \frac{20}{2} = 10$

asked for term

if asked for coefficient, just say 720

1b. *[5 marks]*

Hence or otherwise, find the term in x^3 in the expansion of $(2x+3)^5$.