5. Given that a = 3i - 5k, b = 2i + 7j and c = i + j + k, find the vector d such that a dot d = -9, b dot d = 11 and c dot d = 6

$d = d_1 i + d_2 j + d_3 k$

$a \cdot d = 3d_1 + (-5)d_2 = -9$

$b \cdot d = 2d_1 + 7d_2 = 11$

$c \cdot d = d_1 + d_2 + d_3 = 6$

14. Let $a = 5i - 3j + 7K$, $b = i + j + \lambda K$. Find λ such that $a+b$ is perpendicular to $a-b$

$$(a+b) = (5+1)i + (-3+1j) + (7+\lambda)K$$

$$(a+b) = 6i - 2j + (7+\lambda)K$$

$$(a-b) = 4i - 4j + (7-\lambda)K$$

$$0 = (a+b)\cdot(a-b) = (6\times 4) + (-2\times -4) + (7+\lambda)(7-\lambda)$$

$$= 24 + 8 + 49 - \lambda^2$$

$$0 = 81 - \lambda^2$$

$$\lambda^2 = 81$$

$$\lambda = \pm 9$$

Intersection of Two Lines

Where do 2 vectors cross?

Ex) TWO LINES HAVE EQUATIONS

$$r_1 = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + s\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \text{ and } r_2 = \begin{pmatrix} 6 \\ 2 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 4 \\ 8 \end{pmatrix}$$

SHOW THAT THE LINES INTERSECT AND FIND THE POINT OF INTERSECTION.

* TWO VECTORS ARE EQUAL IF THEIR CORRESPONDING COMPONENTS ARE EQUAL

$$r_1 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + s\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Longrightarrow \begin{aligned} x &= 3 + s \\ y &= s \\ z &= -1 + s \end{aligned}$$

$$r_2 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 4 \\ 8 \end{pmatrix} \Longrightarrow \begin{aligned} x &= 6 \\ y &= 2 + 4t \\ z &= 8t \end{aligned}$$

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$3+s=6$

$s=3$

$-1+s=8\left(\frac{1}{4}\right)$

$-1+s=2$

$s=3$

$s=2+4t$

$3=2+4t$

$1=4t$

$t=1/4$

$-1+s=8t$

$-1+3=8t$

$2=8t$

$t=1/4$

* SINCE THE VALUE OF s and t IS CONSISTENT FOR ALL THREE EQUATIONS, THE TWO LINES MUST INTERSECT.

Point of Int.

$$r_1=\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}3\\0\\-1\end{pmatrix}+3\begin{pmatrix}1\\1\\1\end{pmatrix}=\begin{pmatrix}3+(3)(1)\\0+(3)(1)\\-1+3(1)\end{pmatrix}=\begin{pmatrix}6\\3\\2\end{pmatrix}$$

$$r_2=\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}6\\2\\0\end{pmatrix}+1/4\begin{pmatrix}0\\4\\8\end{pmatrix}=\begin{pmatrix}6\\3\\2\end{pmatrix}$$

$\therefore$ the point of intersection is $(6,3,2)$