1. $r_1 = 4i + 2j + \lambda(2i - 4j) = (4 + 2\lambda)i + (2 - 4\lambda)j$

$r_2 = 11i + 16j + \mu(i + 2j) =$

$r_1 = 4i + 2j + 2\lambda i - 4\lambda j = (4 + 2\lambda)i + (2 - 4\lambda)j$

$r_2 = 11i + \mu i + 16j + 2\mu j = (11 + \mu)i + (16 + 2\mu)j$

$i \quad 4 + 2\lambda = 11 + \mu \Longrightarrow \mu = 2\lambda - 7$

$j \quad 2 - 4\lambda = 16 + 2\mu$

$2 - 4\lambda = 16 + 2(2\lambda - 7)$

$2 - 4\lambda = 16 + 4\lambda - 14$

$-8\lambda = 0 \Longrightarrow \lambda = 0$

$\mu = 2(0) - 7$

$\mu = -7$

i

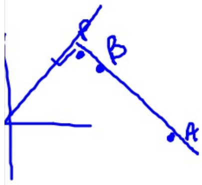
$4 + 2(0) = 4$

or

$11 + (-7) = 4$

$(4, 2)$

7. L : $r = \begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ A : $(5, 7, a)$
B : $(b, 13, -1)$



a) Find the values of a + b

$\begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ a \end{pmatrix} \Longrightarrow$

$6 + t = 5 \Rightarrow t = -1$
$9 + 2t = 7$
$3 - 2t = a \Rightarrow a = 5$

and

$\begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} b \\ 13 \\ -1 \end{pmatrix} \Longrightarrow$

$6 + t = b \Rightarrow b = 8$
$9 + 2t = 13 \Rightarrow t = 2$
$3 - 2t = -1$

b) point P, OP is $\perp$ to L

$\overline{AB} = \begin{pmatrix} 8 \\ 13 \\ -1 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ -6 \end{pmatrix}$

$\overline{AB} = \begin{pmatrix}3\\4\\-6\end{pmatrix}$ $\qquad\qquad$ $\begin{pmatrix}6\\9\\3\end{pmatrix} + t\begin{pmatrix}1\\2\\-2\end{pmatrix}$

$$(\overline{OP})\cdot(\overline{AB}) = 0$$

$$\begin{pmatrix}6+t\\9+2t\\3-2t\end{pmatrix}\cdot\begin{pmatrix}3\\6\\-6\end{pmatrix} = 0$$

c) $\sqrt{4^2+5^2+7^2}$

$= \sqrt{90}$

$= 3\sqrt{10}$

$$3(6+t)+6(9+2t)-6(3-2t)=0$$

$$\cancel{18}+3t+\overset{54}{\cancel{22}}+12t-\cancel{18}+12t=0$$

$$27t+54=0$$

$$t=-2$$

$$\overrightarrow{OP} = \begin{pmatrix}6\\9\\3\end{pmatrix} - 2\begin{pmatrix}1\\2\\-2\end{pmatrix} = \begin{pmatrix}4\\5\\7\end{pmatrix} \qquad P=(4,5,7)$$

12.5 Applications of vectors

The position vector of a boat, A t hours after it leaves the harbor is given by $r_1 = t\begin{pmatrix}30\\15\end{pmatrix}$

A second Boat, B, is passing near the harbor. It's position vector at time t is $r_2 = \begin{pmatrix}50\\5\end{pmatrix} + t\begin{pmatrix}10\\10\end{pmatrix}$ (Km).

a) How far apart are the boats at the time A leaves the harbor?

at $t=0$, Boat A is at the "origin" vector $\begin{pmatrix}0\\0\end{pmatrix}$

and B is at $\begin{pmatrix}50\\5\end{pmatrix}$ $\sqrt{(50-0)^2+(5-0)^2} = \sqrt{2525}$

≈ 50.2 Km

b) How fast is each boat travelling? Km/hr

Need to find the magnitude of the direction vector when $t=1$

A: in 1 hour $\left|1\begin{pmatrix}30\\15\end{pmatrix}\right| = \sqrt{30^2+15^2} = 33.5$ Km/hr

B: in 1 hour $\left|1\begin{pmatrix}10\\10\end{pmatrix}\right| = \sqrt{10^2+10^2} = \sqrt{200} \approx 14.1$ Km/hr

c) Will the boats ever collide?

Can we find a time t where the positions are the same?

x-components: $30t = 50 + 10t$

$\Rightarrow$ $t = 2.5$ hours

y-components: $15t = 5 + 10t$

$\Rightarrow$ $t = 1$ hour

Will never collide

Hw 12L p.437 1, 2, 4