

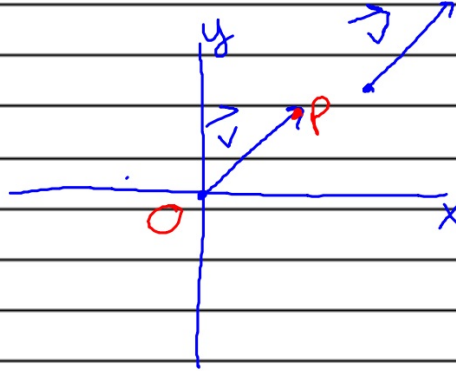
Questions:

Notes:

POSITION VECTORS

9/6/17

A POSITION VECTOR IS ONE WHOSE TAIL HAS BEEN PLACED AT THE ORIGIN - IT IS IN STANDARD POSITION



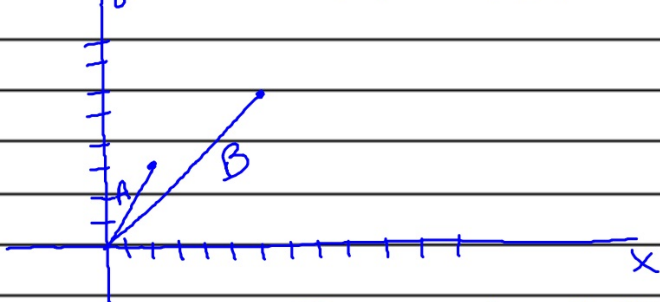
THE POINT P WITH COORDINATES (x, y) HAS POSITION VECTOR

$$\vec{OP} = \begin{pmatrix} x \\ y \end{pmatrix} = xi + yj$$

Questions:

Notes: RESULTANT VECTORS

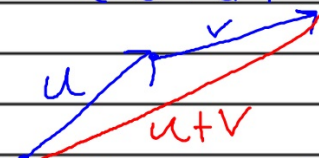
CONSIDER POINTS $A(2,3)$ AND $B(6,6)$
(BOTH AT STANDARD POSITION)



WHAT HAPPENS IF I ADD $A+B$?

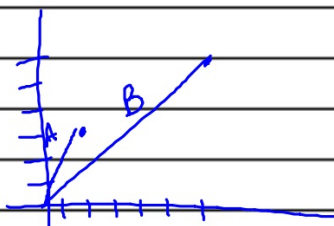
ADDITION OF VECTORS $u = \begin{pmatrix} a \\ b \end{pmatrix}, v = \begin{pmatrix} c \\ d \end{pmatrix}$

Algebra: $u+v = \begin{pmatrix} a+c \\ b+d \end{pmatrix}$

Geometric:  attach tail of v to end of u

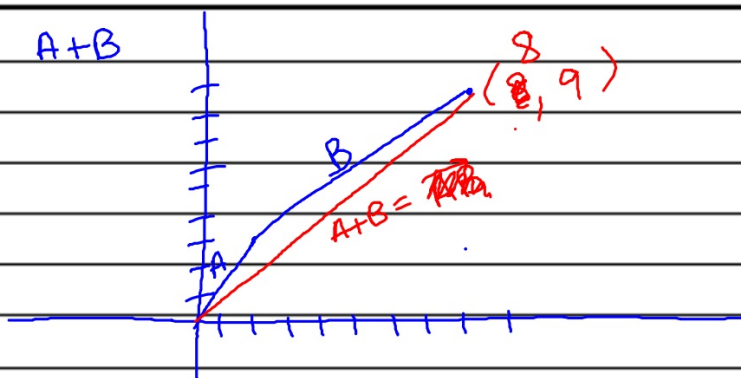
$\vec{u+v}$ is the Resultant Vector

Questions:



Notes:

A+B



$$A+B = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 6 \\ 6 \end{pmatrix} = \begin{pmatrix} 2+6 \\ 3+6 \end{pmatrix} = \begin{pmatrix} 8 \\ 9 \end{pmatrix}$$

You try $A = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$ $B = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

just algebraically $A+B = \begin{pmatrix} 2 \\ 12 \end{pmatrix}$

Questions:

Notes:

SUBTRACTION: $u = \begin{pmatrix} a \\ b \end{pmatrix}$ and $v = \begin{pmatrix} c \\ d \end{pmatrix}$

Algebra: $u - v = \begin{pmatrix} a - c \\ b - d \end{pmatrix}$

Geometric:



make v negative

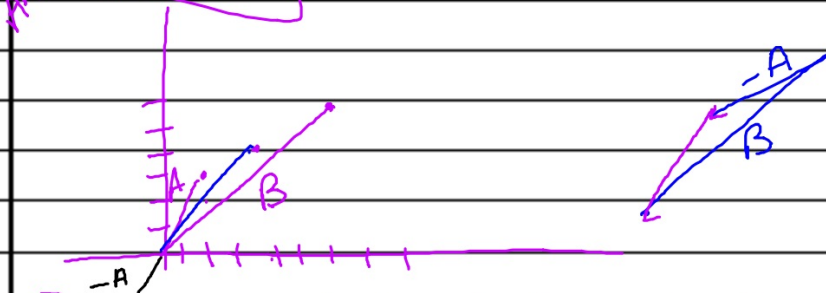
now: connect the endpoints
of u and $-v$

TO FIND THE RESULTANT VECTOR \overline{AB}
BETWEEN TWO POINTS A AND B WE
CAN SUBTRACT THE POSITION VECTOR
A FROM THE POSITION VECTOR B.

Questions:

Notes:

Result $\vec{AB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ From A to B



$$\vec{AB} = \vec{B} - \vec{A} = \begin{pmatrix} 6 \\ 9 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

From O to A, then O to B

$$\text{Thus } \vec{AB} = \vec{OB} - \vec{OA}$$

recall $\vec{AB} = -\vec{BA}$

then

$$\vec{AB} = -\vec{BA} = \vec{OB} - \vec{OA}$$

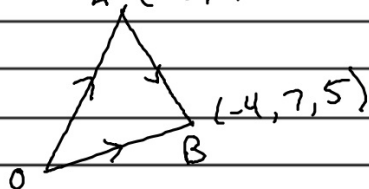
Questions:

Notes:

Ex) Points A and B have coordinates $(-3, 2, 0)$ and $(-4, 7, 5)$

Find \vec{AB}

A $(-3, 2, 0)$



$$\vec{OA} = \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix} \text{ and } \vec{OB} = \begin{pmatrix} -4 \\ 7 \\ 5 \end{pmatrix}$$

$$\vec{AB} = \vec{OB} - \vec{OA} = \begin{pmatrix} -4 \\ 7 \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \\ 5 \end{pmatrix}$$

Ex 2) $\vec{XY} = \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$ and $\vec{XZ} = \begin{pmatrix} 0 \\ -10 \\ -1 \end{pmatrix}$

a. $\vec{YZ} =$

$$\vec{YZ} = \vec{XZ} - \vec{XY} = \begin{pmatrix} 0 \\ -10 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ -11 \\ 2 \end{pmatrix}$$

Questions:

Notes: Practice from 12C

#1. P has coordinates (7,4) and Q (2,3)

Find \vec{PQ} and \vec{QP}

$$\vec{PQ} = \vec{Q} - \vec{P} = \vec{OQ} - \vec{OP}$$

$$= \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 7 \\ 4 \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \end{pmatrix}$$

in terms of i and j

$$\vec{OQ} - \vec{OP} = (2-7)i + (3-4)j = -5i - j$$

$$\vec{P} = 7i + 4j$$

$$\vec{Q} = 2i + 3j$$

TRY: Find $\vec{QP} = \begin{pmatrix} 7 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$$= \begin{pmatrix} 7 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 5i + j$$

Questions:

Notes:

#3 write in $ai+bj+ck$

a. \vec{OP} where P is $(2, -3, 5)$

$$\vec{OP} = 2i - 3j + 5k$$

b. the vector joining $(3, -1, 4)$ to the origin (asking for \vec{RO})

Origin - vector

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = -3i + j - 4k$$

c. the vector from $(3, -1, 4)$ to $(1, 2, 5)$

$$= \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$$

Hw 12c p. 416 #2-6

Questions:

Notes:

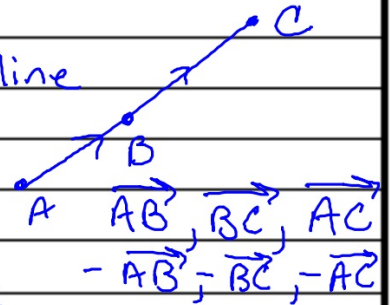
Ex) $\vec{A} = 3\vec{i} + 2\vec{j} - 3\vec{k}$
 $\vec{B} = 2\vec{i} + \vec{j} - 5\vec{k}$

find $\vec{BA} = \vec{OA} - \vec{OB}$
 $= (3-2)\vec{i} + (2-1)\vec{j} + (-3-(-5))\vec{k}$
 $= \vec{i} + \vec{j} + 2\vec{k}$

COLINEAR VECTORS

vectors on the same line

- If two or more vectors are on the same line (colinear), they will be parallel, and be scalar multiples of each other



Questions:

Notes:

Ex Show that A, B, C with position vectors $i - 2j + 3k$, $-2i + 3j - k$, $4i - 7j + 7k$ are colinear

Plan: start by finding the vector joining any 2 of the points
ie. show that \vec{AB} and \vec{BC} are scalar multiples of each other \Rightarrow parallel

HW 12D
p. 417 #1-3

$$\vec{AB} = \vec{OB} - \vec{OA} = \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ -4 \end{pmatrix} \\ = -3i + 5j - 4k$$

now repeat with any other two vectors points

$$\vec{BC} = 6i - 10j + 8k$$

$$\vec{AB} = -\frac{1}{2}\vec{BC} \Rightarrow \vec{AB} \parallel \vec{BC}$$

Since they are scalar multiples of each other. And, since they both contain the point B, they are colinear

