



Essential Question:

How CAN WE USE LOGS + EXPONENTS TO  
 SOLVE REAL-WORLD PROBLEMS?

Questions:

Notes:

REAL-WORLD APPLICATIONS

(EXPONENTIAL GROWTH/DECAY)

- CARBON DATING
- INTEREST (INVEST/LOANS)
- RADIOACTIVITY
- CANCER CELL GROWTH
- BACTERIA
- ANIMAL POPULATIONS
- VIRAL VIDEOS

EX) AT 12:00, A SINGLE BACTERIA  
 COLONIZES A CAN. THE GROWTH  
 OF THE POPULATION CAN BE MODELED  
 WITH THE FUNCTION  $P(t) = 2^t$ ,  
 WHERE  $t$  IS TIME IN MINUTES SINCE  
 12:00.

a) HOW MANY BACTERIA ARE IN  
 THE CAN AT 12:05?

$$t = 5$$

$$P(5) = 2^5 = 32$$

p. 1. 2. 3. 4. 5.

b) THE CAN IS COMPLETELY FULL  
 AT 1:00. AT WHAT TIME WAS THE  
 CAN  $\frac{1}{2}$  FULL?

FULL } took 60 minutes to fill

$$P(60) = 2^{60} = 1.153 \times 10^{18}$$

$\frac{1}{2}$  of  $\nearrow$  is  $5.77 \times 10^{17}$

$$2^t = 5.77 \times 10^{17}$$

$$\ln 2^t = \ln 5.77 \times 10^{17}$$

$$t \frac{\ln 2}{\ln 2} = \frac{\ln 5.77 \times 10^{17}}{\ln 2} \approx 59$$



Questions:	Notes: <u>Ex) COMPOUND INTEREST</u>
	$A(t) = P \left(1 + \frac{r}{n}\right)^{nt}$ <p>A = amount after t years  r = rate, in decimals  t = number of years  n = number of times per year of compounding  P = PRINCIPAL</p>
	<p>CONTINUOUS COMPOUNDING</p> $A = P e^{rt} \quad (\text{Pert})$ <p>COMPARE THE INVESTMENT OF P DOLLARS  AT: INVESTED FOR 1 YEAR</p> <p>a) 8 1/2% PER YEAR COMPOUNDED QUARTERLY</p> <p>b) 8% PER ANNUM COMPOUNDED CONTINUOUSLY</p> <p>WHICH IS THE BETTER INVESTMENT?</p> <p>a) <math>A(t) = P \left(1 + \frac{r}{n}\right)^{nt}</math>  <math>r = .085, t = 1, n = 4</math></p> <p>b) <math>A(t) = P e^{rt} = P 1.08329</math></p>
	<p>How MANY YEARS WOULD AN INVESTMENT AT 8% COMPOUNDED DAILY WOULD IT TAKE TO RETURN \$1.90 FOR EVERY DOLLAR INVESTED?</p> <p><math>A(t) = P \left(1 + \frac{r}{n}\right)^{nt}</math>     <math>n = 365</math></p> <p><math>1.90 = 1 \left(1 + \frac{.08}{365}\right)^{365t}</math></p> <p><math>\ln 1.90 = \ln \left(1 + \frac{.08}{365}\right)^{365t}</math></p>

better investment →

$$15 = 2^x$$

$$\ln 15 = \ln 2^x$$

$$\ln 15 = x \ln 2$$

$$\frac{\ln 15}{\ln 2} = x$$

Process example →

HW 4T  
P. 133  
#1, 2

$$\ln 1.90 = 365t \ln \left(1 + \frac{.08}{365}\right)$$

$$\ln 1.90 = 365 \ln \left(1 + \frac{.08}{365}\right) t$$

$$\frac{\ln 1.90}{365 \cdot \ln \left(1 + \frac{.08}{365}\right)} = t$$

$$t \approx 8.02 \text{ years}$$

Questions:	Notes:
$P\left(1 + \frac{r}{n}\right)^{nt}$ $P(1 + .10)^d$	<p>Ta      <del>Day</del> 100</p> <p>Day 1    100 x 1.1      100 + 100 x .1</p> <p>i. → Day 2    100 x 1.1<sup>2</sup>      100 + 100 x .1</p> <p>ii after 1 week = 100 x 1.1<sup>7</sup></p> <p><math>f(x) = a^x</math></p>
$\ln \square = \ln \square$	<p>POPULATION GROWTH</p> <p><math>P = P_0(1+r)^t</math>      t in units</p> <p><math>P = 100(1+.10)^t</math>      given</p> <p><math>P = P_0 e^{rt} \rightarrow \ln \frac{P}{P_0} = \ln e^{rt}</math></p>
	<p>b. How long for 250 infected?</p> <p><math display="block">\frac{250}{100} = \frac{100 \times 1.1^d}{100}</math></p> <p><math display="block">2.5 = 1.1^d</math></p> <p><math display="block">\ln 2.5 = \ln 1.1^d</math></p> <p><math display="block">\ln 2.5 = d \ln 1.1</math></p> <p><math display="block">\frac{\ln 2.5}{\ln 1.1} = d</math></p>

$d \approx 9.6$  days  
10 days

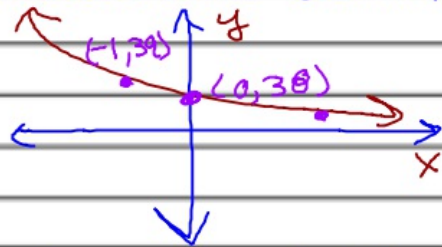
Questions:

Notes: p.134 #4

$$v = 9 + 29e^{-0.063t}$$

a. Sketch graph of  $s$  against  $t$   
(Distance versus time graph)

-1	39
0	38
1	36.229



b. What was his speed the instant the parachute opened?

Time  $t=0$

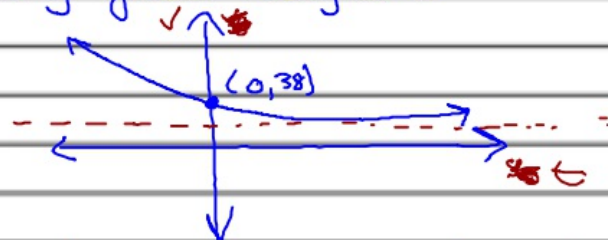
$$v = 9 + 29e^{-0.063(0)}$$

$$v = 9 + 29e^0$$

$$v = 9 + 29$$

$$v = 38 \text{ ms}^{-1}$$

c. What is his lowest possible speed if he fell from a very great height?



as  $t \rightarrow \infty$ ,  $v \rightarrow 9 \text{ ms}^{-1}$   
↑  
approaches

d. If he landed after 45s, what is his speed on landing?

$$v = 9 + 29e^{-0.063(45)}$$

$$v = 10.7 \text{ ms}^{-1}$$

Questions:	Notes:
	e. How <u>long</u> did it take him to
	reach <u>1/2</u> the speed he had
	when the parachute opened?
	Initial speed was $38 \text{ ms}^{-1}$
	so $1/2$ that is $19 \text{ ms}^{-1}$
HW 4T 3,5	$9 + 29e^{-.063t} = 19$
p.134	$29e^{-.063t} = 10$
Review	$e^{-.063t} = \frac{10}{29}$
Exercises	$-.063t = \ln\left(\frac{10}{29}\right)$
all	$t = \frac{\ln\left(\frac{10}{29}\right)}{-.063}$
p.134-136	$t \approx 16.9 \text{ s}$
	$t \approx 17 \text{ s}$