

$$\begin{aligned}3b \quad \sin(2x) &= \cos(2x) \\ \sin(2x) - \cos(2x) &= 0 \\ \sin(2x) - (1 - 2\sin^2x) &= 0 \\ \sin(2x) - 1 + 2\sin^2x &= 0 \\ 2\sin^2x + \sin(2x) - 1 &= 0 \\ 2\sin^2x + 2\sin x \cos x - 1 &= 0 \\ 2\sin x (\sin x + \cos x) - 1 &\end{aligned}$$

$$3d. \sin(4x) = \sin(2x)$$

$$\frac{2\sin(2x)\cos(2x)}{\sin(2x)} = \frac{\sin(2x)}{\sin(2x)}$$

~~$4\sin x \cos x \cos(2x) = \sin(2x)$~~

$$2\cos(2x) = 1$$

$$\cos(2x) = \frac{1}{2}$$

$$2x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{12}$$

$$0 \leq x \leq \pi$$
$$0 \leq 2x \leq 2\pi$$

$$2\sin(2x)\cos(2x) - \sin(2x) = 0$$

$$\sin(2x)(2\cos(2x) - 1) = 0$$

$$\sin(2x) = 0$$

$$2x = 0, \pi, 2\pi$$

$$x = 0, \frac{\pi}{2}, \pi$$

$$5d \quad \sin^2 x + 2\sin x \cos x + \cos^2 x = 1 + \sin 2x$$

$$5d \quad \cancel{\cos^2} \cos \theta + \sin \theta = \frac{1 - 2\sin^2 \theta}{\cos \theta - \sin \theta} \quad \text{cos}(2\theta)$$

$$\begin{aligned} \text{RHS} &= \frac{\cos^2 \theta - \sin^2 \theta}{\cos \theta - \sin \theta} \quad \text{cos}(2\theta) \\ &= \frac{(\cos \theta - \sin \theta)(\cos \theta + \sin \theta)}{(\cos \theta - \sin \theta)} \quad \text{diff of squares} \\ &= \cos \theta + \sin \theta \\ &= \text{LHS} \quad \checkmark \end{aligned}$$