

4.6 Laws of Logarithms

$$\text{I. } x = a^p \text{ and } y = a^q \implies p = \log_a x \text{ and } q = \log_a y$$

then $xy = a^p \cdot a^q = a^{p+q}$

$$\text{so that } \log_a xy \\ = \log_a a^{p+q} = p+q$$

$$\therefore \log_a xy = \log_a x + \log_a y$$

Ex) write $\log_2(x+1) + \log_2(5)$ as $1 \log$
 $\log_2(5(x+1))$
 $\log_2(5x+5)$

$$\text{II. } \log_a \frac{x}{y} = \log_a x - \log_a y$$

$$\text{III. } \log_a x^n = n \log_a x$$

$$\text{IV. } \log_a \frac{1}{x} = \log_a x^{-1} = -\log_a x$$

Ex) Express $\log_2 5 + \frac{1}{2} \log_2 36 - \log_2 10$ as 1 log.

* 1 piece at a time

$$\log_2 5 + \underbrace{\log_2 36^{\frac{1}{2}}}_{\log_2 6} - \log_2 10$$
$$\log_2(5 \cdot 6) - \log_2 10$$
$$\log_2 \left(\frac{30}{10} \right) = \log_2 3$$

Ex] Express as a single log

$$\log(x) - \log(y) - \log(z)$$
$$\log(x) - (\log(y) + \log(z))$$
$$\log(x) - \log(yz)$$
$$\log\left(\frac{x}{yz}\right)$$

HW 4m p. 124
1, 2, 3 (a, c, d, e)