1.6 Transforming Functions

Translations

Shifting up or down:

$f(x) + K$ translates f vertically K units

↑ outside ( ) $\Rightarrow$ vertical $+ \uparrow$ $- \downarrow$

$f(x + K)$ translates f left K units

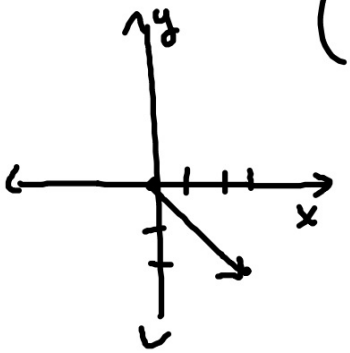
$f(x - K)$ translates f right K units

↑ (inside) change is $\leftrightarrow$ and opposite what it looks like

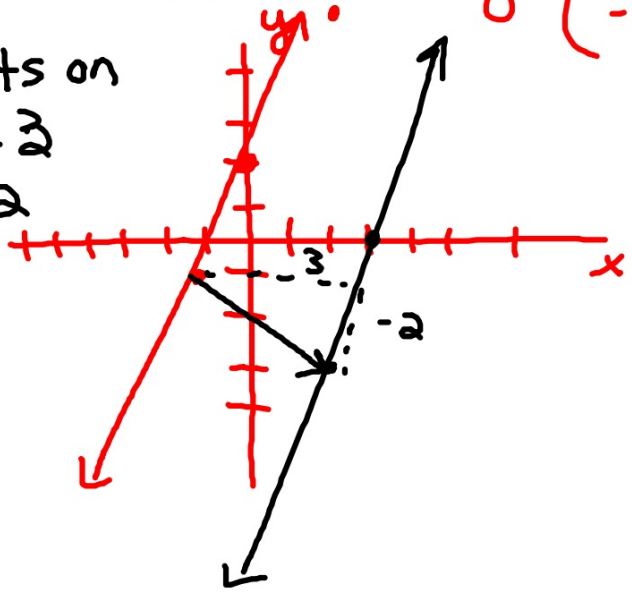
translation vector

translations can be represented by vectors in the form $\begin{pmatrix} a \\ b \end{pmatrix}$ where a is the horizontal (x) and b is the vertical (y).

Ex) translate $f(x) = 3x + 2$ by $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$



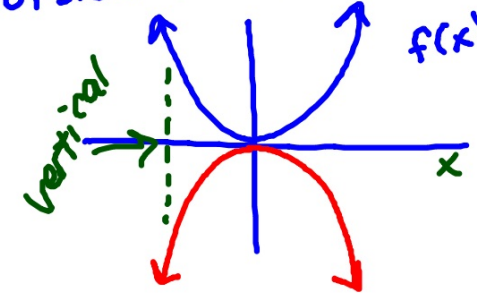
$\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ all points on f go right 3 and down 2



Reflections

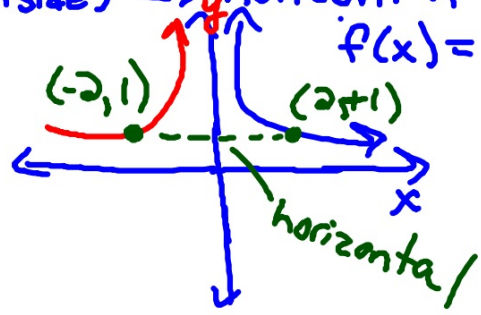
$-f(x)$ reflects $f(x)$ about (or in) the x-axis

outside ( ) => vertical change



$f(-x)$ reflects $f(x)$ about/over/in the y-axis

(inside) => horizontal



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Stretches/compressions

$p\,f(x)$ stretches or compresses $f(x)$ by a factor of p.

outside $\Rightarrow$ vertical change

$f(qx)$ stretches or compresses $f(x)$ by a factor of $\frac{1}{q}$

(inside) $\Rightarrow$ horizontal and backwards what it looks like

ex) $f(x) = (3x)^3$ is a compression of $\frac{1}{3}$

In which order do I graph transformations of functions?

- Vertical Shifts.
- Horizontal Shifts.
- Reflection about the x-axis.
- Reflection about the y-axis.
- Vertical ~~shifting~~ compress or stretching.
- Horizontal shifting shrink or stretching.

Ex $f(x) = -7(-3(x+2))^3 + 4$

- up 4
- Right 6 $(-3x-6)^3$
- reflect over x-axis
- reflect over y
- Stretch ↕ by factor of 7
- shrink ↔ by $\frac{1}{3}$

Ex 3 functions q, s and t are transformations of f(x). write each transformation in terms of f(x).

$f(x) = |x|$

s(x) - horizontal shift left 4 units

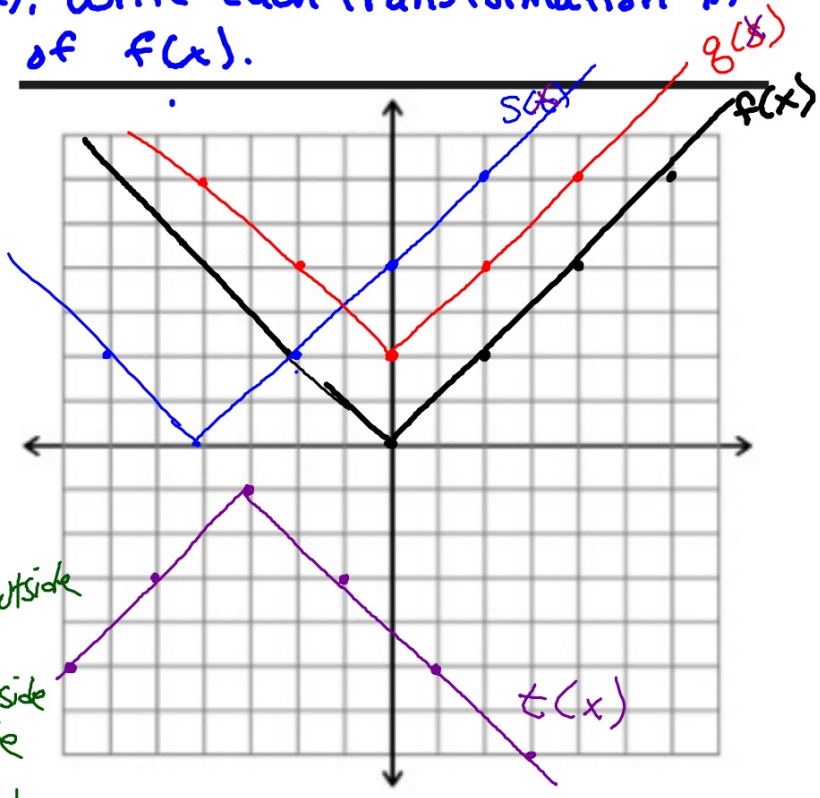
$s(x) = |x+4|$

q(x) - vertical shift up 2 units

$q(x) = |x| + 2$

t(x) = reflect over x (vertical) } outside
left 3 units - inside
down 1 - outside

$t(x) = -|x+3| - 1$



Ex) DESCRIBE THE CHANGE THAT TAKES $f(x)$ to $g(x)$

$f(x) = x^2$; $g(x) = -(2x)^2 - 4$

- vertical reflection over x-axis
- horizontal shrink by a factor of $\frac{1}{2}$
- vertical translation (shift) down 4