3-3 Sample space diagrams and the Product Rule

- it is possible to list all possible outcomes (if there aren't <u>too</u> many)

<u>Ex)</u> A couple wants to have 3 children
List all possible outcomes

BGB GGG
BBB GGB BGG

∴ order matters

GGG BBB
GGB BGB
GBG BGG
GBB BBG

} sample space

Example 7

A fair spinner with the numbers 1, 2 and 3 on it as shown is spun three times. List all the possible outcomes from this experiment.
Hence find the probability that the score on the last spin is greater than the scores on the first two spins.

Answer

The 27 outcomes are:

1 1 1	1 2 1	1 3 1
1 1 2	1 2 2	1 3 2
1 1 3	1 2 3	1 3 3
2 1 1	2 2 1	2 3 1
2 1 2	2 2 2	2 3 2
2 1 3	2 2 3	2 3 3
3 1 1	3 2 1	3 3 1
3 1 2	3 2 2	3 3 2
3 1 3	3 2 3	3 3 3

Of these, the five in red have the score on the last spin greater than the scores on the first two spins.

Hence the probability is $\frac{5}{27}$.

When listing all the outcomes, you need to be systematic so that you do not miss any out.

P. 77

$P(E) = \frac{5}{27}$

X-2.3

Sample Space diagram - another option for organizing outcomes.

Ex: list all possible outcomes of rolling 2 dice

P(sum is perfect square)

P(4, 9)

die 1

die 2	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

36 poss outcomes

$P(E) = \frac{7}{36}$

Product Rule for Independent Events

<u>def</u> Two (or more) events are said to be <u>independent</u> if the outcome of one is not influenced by the other.

$$P(A \cap B) = P(A) \cdot P(B)$$

Ex) Roll 1 die, toss 1 coin

Sample Space

	1	2	3	4	5	6
H	(H,1)	(H,2)	(H,3)	(H,4)	(H,5)	(H,6)
T	(T,1)	(T,2)	(T,3)	(T,4)	(T,5)	(T,6)

$P(H) = \frac{6}{12} = \frac{1}{2}$

$P(\text{die roll less than } 3) = \frac{4}{12} = \frac{1}{3}$

$P(\text{Heads and roll less than } 3) = \frac{2}{12} = \frac{1}{6}$

Product Rule $\quad P(H) \cdot P(<3) = P(H \cap <3)$

$\left(\frac{1}{2}\right) \cdot \left(\frac{1}{3}\right) = \frac{1}{6}$

Ex) A bag contains 3 red and 2 white balls
Another bag contains 1 red and 4 white balls.
A ball is selected from each bag. Find the probability that:

a) both are red $P(R_1)=\frac{3}{5}$ $P(R_2)=\frac{1}{5}$

and

$\cap$

$$P(R_1 \cap R_2) = \frac{3}{5}\cdot\frac{1}{5} = \frac{3}{25}$$

b) balls are diff.

$P(R_1)=\frac{3}{5}$, $P(W_1)=\frac{4}{5}$ $P(R_1 \cap W_2)=\frac{3}{5}\times\frac{4}{5}=\frac{12}{25}$

$P(W_1)=\frac{2}{5}$ and $P(R_2)=\frac{1}{5}$ $P(W_1 \cap R_2)=\frac{2}{25}$

c) at least one is white

$$\frac{12}{25}+\frac{2}{25}=\frac{8}{25}$$

HW 3E p79 #1-5