5-3 Rational Functions

def A rational function is a function of the form

$$f(x) = \frac{g(x)}{h(x)} = \frac{a_n x^n + \cdots\cdots + K}{b_m x^m + \cdots\cdots + l}$$

g is a polynomial of degree n with a leading coefficent of a

h is a polynomial of degree m with a leading coefficient of b.

(Some) Properties of Rational Functions

x-intercepts: occur when $y=0$; $f(x)=0$
and solve for x

y-intercepts: occur when $x=0$; $f(0)$
plug 0 in for x

Asymptotes: imaginary line at which a function
will approach, but never touch
(except occassional horiz. asymp)

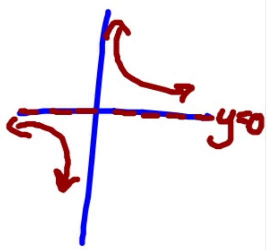
1. vertical: occur where the function is undefined
(zero in denominator)
2. Horizontal
3. Slant

$$f(x)=\frac{a_n x^n + \ldots}{b_m x^m + \ldots}$$

For slant and horizontal asymptotes, the key is the degree (and leading coefficients)

if $\overset{\text{top}}{n} < \overset{\text{bottom}}{m}$ then $f(x)$ has a horizontal asymptote at $y=0$

Ex: $y=\frac{1}{x}=\frac{x^0}{x^1}$ $\quad 0<1 \; \therefore$



if $n=m$ then $f(x)$ has a horizontal asymtote at $y=\frac{a}{b}$

Ex: $y=\frac{3x^4+\ldots}{5x^4+\ldots}$ $\quad$ asymp: $y=\frac{3}{5}$

if $n>m$ then $f(x)$ has a slant asymptote. Divide $g(x)$ by $h(x)$ to find the equation of the asymptote

Ex for the function $y = \frac{x+1}{2x-4}$

a) Sketch the graph

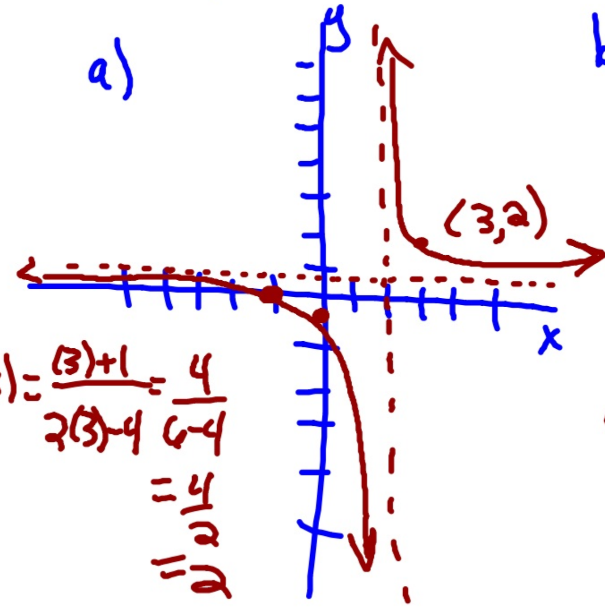
b) find the asymptotes

c) State Domain + Range

D: $\{x \mid x \neq 2\}$

R: $\{y \mid y \neq \frac{1}{2}\}$

a)



$f(3) = \frac{(3)+1}{2(3)-4} = \frac{4}{6-4}$

$= \frac{4}{2}$

$= 2$

b) vert: when denom = 0

$2x-4=0$

$2x=4$

$x=2$ *equation

horiz: $n=m \Rightarrow y=\frac{a}{b}$

$y=\frac{1}{2}$

x-int: $y=0$

$(2x-4)\cdot 0 = \frac{x+1}{\cancel{2x-4}}\,\cancel{(2x-4)}$

$0 = x+1$

$-1 = x$

y-int: $x=0 \Rightarrow y=\frac{1}{-4}$