4.8 APPLICATIONS OF LOGARITHMIC AND EXPONENTIAL FUNCTIONS

EXPONENTIAL GROWTH AND DECAY

- BACTERIA GROWTH
- ANIMAL POPULATIONS
- COMPOUND INTEREST
- HEAT TRANSFER
- HALF-LIFE

Ex 1: At 12:00, a single bacteria colonizes a can. The growth of the population can be modeled with the function

$$P(t) = 2^t$$

where *t* is time in minutes since 12:00.

a) How many bacteria are in the can at 12:05?

$$P(5) = 2^5 = 32$$

At 12:10?

$$P(10) = 2^{10} = 1024$$

b) The can is completely full at 1:00. At what time was the can 1/2 full of bacteria?

At 1:00, it's been 60 minutes

so, $2^{60} = 1.153 \times 10^{18}$

$\frac{1}{2}$ of $1.153 \times 10^{18} = 5.77 \times 10^{17}$

then $2^t = 5.77 \times 10^{17}$

$\cancel{\log_2} 2^t = \log_2 5.77 \times 10^{17}$

$t \approx 59$ minutes

Ex Compound Interest $A(t)=P(1+\frac{r}{n})^{nt}$

P = Principle

r = rate, in decimals

n = number of times per year interest is compounded

t = # of years

Continuously Compounded Interest

$\cancel{P=e^{rt}}$ $A=Pe^{rt}$

Compare the investment of $P At

a) 8½% per year compounded Quarterly

$r=0.085$ $A(1)=P\left(1+\frac{.085}{4}\right)^{4\cdot 1} \approx 1.088P$

$n=4$

$t=1$

to 8% per year compounded continuously

$$A=Pe^{0.08(1)}=1.083P$$

t How many years would an investment at 8%, compounded daily, take to return \$1.90 on a \$1 Investment?

$r=0.08$, $n=365$, $(A(t))$, $P=1$

$$A(t)=P\left(1+\frac{r}{n}\right)^{nt}$$

$$1.90=1\left(1+\frac{0.08}{365}\right)^{365t}$$

$$\ln 1.90 = 365t\ln\left(1+\frac{0.08}{365}\right)$$

$$365t=\frac{\ln 1.90}{\ln\left(1+\frac{0.08}{365}\right)}$$

$$t=\frac{\ln 1.90}{365\ln\left(1+\frac{0.08}{365}\right)}$$

$$t\approx 8.02 \text{ yrs}$$

HW 4T #1235