Pascal's Triangle

row 0 →	1	1	2^0
1	1 1	2	2^1
2	1 2 1	4	
3	1 3 3 1	8	
4	1 4 6 4 1	16	
5	1 5 10 10 5 1	32	
6	1 6 15 20 15 6 1		
7	1 7 21 35 35 21 7 1		
8	1 8 28 56 70 56 28 8 1		

Investigation:

Expand each polynomial:

$(a+b)^0 = 1$

$(a+b)^1 = a+b$

a) $(a+b)^2 = a^2 + 2ab + b^2$

b) $(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$

c) $(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$

d) $(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

1 6 15 20 15 6 1

e) $(a+b)^6 = 1a^6 + 6a^5b^1 + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + 1b^6$

Patterns:

- coefficients from Pascal's $\triangle$
- a s, descending powers match row
- bs, ascending to match row
- exponents on a + b add to row

The Binomial Theorem states that for any power of a binomial, $n \in \mathbb{N}$ ← Natural #s

the # of the element

is a member of

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \cdots$$

or

$$(a+b)^n = \sum_{r=0}^{n}\left[\binom{n}{r}(a)^{n-r}(b^r)\right]$$