Rationalizing

2 b
4 d - C

Anything that is <u>not</u> rational.

rational: <u>any</u> # that can be written as: $\frac{\text{whole \#}}{\text{whole \#}}$

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{4}} = \frac{\sqrt{2}}{2}$$

To rationalize the denominator - multiply by 1

$$\sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$$

$$\tan(30^\circ) = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \cdot \frac{\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}} = \frac{\frac{\cancel{2}}{\cancel{2}\sqrt{3}}}{1}$$

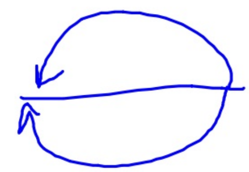
$\frac{\pi}{6}$

$$\frac{1}{2} \cdot \frac{\cancel{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{3}$$

4d. $-\pi \le x \le \pi$



$$4\cos^2 x + 2 = 5$$

$$\frac{4}{4}\cos^2 x = \frac{3}{4}$$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \pm\sqrt{\frac{3}{4}}$$

$$\cos x = \pm\frac{\sqrt{3}}{2}$$

$$x = \cos^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)$$

$\pm\frac{\pi}{6}, \pm\frac{5\pi}{6}$

$$x = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$$

2b) $\tan(3\theta) = 1$

2d

$-3\pi \le 3\theta \le 3\pi$

$3\theta = \frac{-11\pi}{4}, \frac{-3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}$

$\theta = \frac{-11\pi}{12}, -\frac{3\pi}{12}, \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}$

$-\frac{\pi}{4}$ $\frac{3\pi}{4}$ $-\frac{12\pi}{4} \le \frac{12\pi}{4} \le \frac{12\pi}{4}$

2c) $\cos^2\left(\frac{\theta}{2}\right) = \frac{1}{2}$ let $x = \frac{\theta}{2}$

$-\frac{\pi}{2} \le \frac{\theta}{2} \le \frac{\pi}{2}$

$\cos^2(x) = \frac{1}{2}$

$\cos(x) = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$

$x = \pm\frac{\pi}{4}$

$\cancel{2} \cdot \frac{\theta}{\cancel{2}} = \pm\frac{\pi}{4} \cdot 2$

$\theta = \pm\frac{\pi}{2}$

Trig Identities

① Pythag. Identities

$$\sin^2\theta + \cos^2\theta = 1$$

② Double Angle Identities

- for sine

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

— by solving $\cos 2(2\theta) = 1 - \sin 2\theta$ for sine

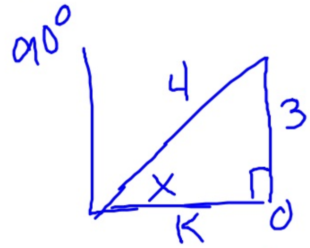
- for cosine

$$\cos(2\theta) = 1 - 2\sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= \cos^2\theta \sin^2\theta$$

p. 457 — comes from the Law of Cosines

Ex) If $\sin x = \frac{3}{4}$, and $0^\circ \le x \le 90^\circ$

a) $\cos x$ b) $\sin(2x)$ c) $\cos(2x)$ d) $\tan(2x)$



a) $\cos(x) = \frac{\sqrt{7}}{4}$

b) $\sin(2x)$

$= 2\sin x \cos x$

$= 2\left(\frac{3}{4}\right)\left(\frac{\sqrt{7}}{4}\right)$

$= \frac{6\sqrt{7}}{16} = \frac{3\sqrt{7}}{8}$

$K^2 + 3^2 = 4^2$

$K^2 + 9 = 16$

$K^2 = 7$

$K = \pm\sqrt{7}$

HW 13E
p. 460
1, 4, 5, 6

c) $\cos(2x) = 1 - 2\sin^2(x)$

$= 1 - 2\left(\frac{3}{4}\right)^2 = 1 - \frac{9}{8} = -\frac{1}{8}$

d) $\tan(2x) = \frac{\sin(2x)}{\cos(2x)} = \frac{\frac{3\sqrt{7}}{8}}{-\frac{1}{8}} = \frac{-3\sqrt{7}}{1}$