6-6 Geometric Series

Just as an arithmetic series is the sum of the terms of an arithmetic sequence, a geometric series is the sum of the terms of a geometric sequence.

$$S_n = u_1 + u_1 r + u_1 r^2 + \cdots + u_1 r^{n-2} + u_1 r^{n-1}$$

(mathmagic happens see p. 195 for details)

You can find the sum of the first n terms of a geometric series using the formula:

$$S_n = \frac{u_1(1-r^n)}{1-r}, \text{ where } r \neq 1$$

Ex1) Calculate the first 12 terms of the series $1+3+9+\cdots$

$u_1 = 1$ $\quad r = \frac{u_2}{u_1} = \frac{3}{1} = 3$

$$S_{12} = \frac{1(1-3^{12})}{1-3} = 265,720$$

Ex2) Calculate the value of S_{20} for the series $3-6+12-24+\cdots\cdots$ (alternating sequence)

$u_1 = 3,\ r = -2$

$$S_{20} = \frac{3(1-(-2)^{20})}{1-(-2)}$$

$$S_{20} \approx -1048575$$

$\boxed{\text{Ex } 3}$ Find the # of terms in the series

$$8192 + 6144 + 4608 + \cdots + \underbrace{1458}_{u_n}$$

$u_1 = 8192$ $\quad r = 0.75$ or $\frac{3}{4}$

$u_n = u_1 r^{n-1}$

(the n^{th} term of a geo sequence)

$$\frac{1458}{8192} = \frac{\cancel{8192}(0.75)^{n-1}}{\cancel{8192}}$$

$$\frac{1458}{8192} = 0.75^{n-1}$$

$$\ln\left(\frac{1458}{8192}\right) = \ln(0.75)^{n-1}$$

$$\ln\left(\frac{1458}{8192}\right) = (n-1)\ln(0.75)$$

$$\frac{\ln\left(\frac{1458}{8192}\right)}{\ln(0.75)} = n-1 \qquad n = 7$$

Ex 4) using a calculator, determine the least value of n for which $S_n > 500$ in the geometric series

$$3 + 3\sqrt{2} + 6 + 6\sqrt{2} + \cdots +$$

$u_1 = 3 \qquad r = \frac{3\sqrt{2}}{3} = \sqrt{2}$

$$S_n = \frac{3(1 - \sqrt{2}^{\,n})}{\sqrt{2} - 1} > 500$$

$n > 12$

or $n = 13$

HW 6I p.176 1ad, 2bd, 3

6J p.178 1ac, 2, 3, 4