New test - January 16, 2017 *[92 marks]*

1a. Two standard six-sided dice are tossed. A diagram representing the sample space is shown below. *[6 marks]*

score on second die

		1	2	3	4	5	6
	1	•	•	•	•	•	•
	2	•	•	•	•	•	•
score on first die	3	•	•	•	•	•	•
	4	•	•	•	•	•	•
	5	•	•	•	•	•	•
	6	•	•	•	•	•	•

Let X be the sum of the scores on the two dice.

(i) Find $\mathrm{P}(X = 6)$.

(ii) Find $\mathrm{P}(X > 6)$.

(iii) Find $\mathrm{P}(X = 7 | X > 6)$.

Markscheme

(i) number of ways of getting $X = 6$ is 5 *A1*

$\mathrm{P}(X = 6) = \frac{5}{36}$ *A1 N2*

(ii) number of ways of getting $X > 6$ is 21 *A1*

$\mathrm{P}(X > 6) = \frac{21}{36} \left(= \frac{7}{12}\right)$ *A1 N2*

(iii) $\mathrm{P}(X = 7 | X > 6) = \frac{6}{21} \left(= \frac{2}{7}\right)$ *A2 N2*

[6 marks]

Examiners report

[N/A]

1b. Elena plays a game where she tosses two dice. *[8 marks]*

If the sum is 6, she **wins** 3 points.

If the sum is greater than 6, she **wins** 1 point.

If the sum is less than 6, she **loses** k points.

Find the value of k for which the game is fair.

Markscheme

attempt to find $\mathrm{P}(X < 6)$ *M1*

e.g. $1 - \frac{5}{36} - \frac{21}{36}$

$\mathrm{P}(X < 6) = \frac{10}{36}$ *A1*

fair game if $\mathrm{E}(W) = 0$ (may be seen anywhere) *R1*

attempt to substitute into $\mathrm{E}(X)$ formula *M1*

e.g. $3\left(\frac{5}{36}\right) + 1\left(\frac{21}{36}\right) - k\left(\frac{10}{36}\right)$

correct substitution into $\mathrm{E}(W) = 0$ *A1*

e.g. $3\left(\frac{5}{36}\right) + 1\left(\frac{21}{36}\right) - k\left(\frac{10}{36}\right) = 0$

work towards solving *M1*

e.g. $15 + 21 - 10k = 0$

$36 = 10k$ *A1*

$k = \frac{36}{10}\ (= 3.6)$ *A1 N4*

[8 marks]

Examiners report

[N/A]

2a. Two events A and B are such that $\mathrm{P}(A) = 0.2$ and $\mathrm{P}(A \cup B) = 0.5$. *[2 marks]*

Given that A and B are mutually exclusive, find $\mathrm{P}(B)$.

Markscheme

correct approach ***(A1)***

eg $0.5 = 0.2 + \mathrm{P}(B),\ \mathrm{P}(A \cap B) = 0$

$\mathrm{P}(B) = 0.3$ *A1 N2*

[2 marks]

Examiners report

[N/A]

2b. Given that A and B are independent, find $\mathrm{P}(B)$. *[4 marks]*

Markscheme

Correct expression for $\mathrm{P}(A \cap B)$ (seen anywhere) *A1*

eg $\mathrm{P}(A \cap B) = 0.2\mathrm{P}(B),\ 0.2x$

attempt to substitute into correct formula for $\mathrm{P}(A \cup B)$ *(M1)*

eg $\mathrm{P}(A \cup B) = 0.2 + \mathrm{P}(B) - \mathrm{P}(A \cap B),\ \mathrm{P}(A \cup B) = 0.2 + x - 0.2x$

correct working *(A1)*

eg $0.5 = 0.2 + \mathrm{P}(B) - 0.2\mathrm{P}(B),\ 0.8x = 0.3$

$\mathrm{P}(B) = \frac{3}{8}\ (= 0.375,\ \text{exact})$ *A1 N3*

[4 marks]

Examiners report

[N/A]

3a. Samantha goes to school five days a week. When it rains, the probability that she goes to school by bus is 0.5. When it does not rain, the probability that she goes to school by bus is 0.3. The probability that it rains on any given day is 0.2. *[4 marks]*

On a randomly selected school day, find the probability that Samantha goes to school by bus.

Markscheme

appropriate approach *(M1)*

eg $\mathrm{P}(R \cap B) + \mathrm{P}(R' \cap B)$, tree diagram,

one correct multiplication *(A1)*

eg $0.2 \times 0.5,\ 0.24$

correct working *(A1)*

eg $0.2 \times 0.5 + 0.8 \times 0.3,\ 0.1 + 0.24$

$\mathrm{P}(\text{bus}) = 0.34(\text{exact})$ *A1 N3*

[4 marks]

Examiners report

[N/A]

3b. Given that Samantha went to school by bus on Monday, find the probability that it was raining. *[3 marks]*

Markscheme

recognizing conditional probability *(R1)*

eg $\mathrm{P}(A|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

correct working *A1*

eg $\frac{0.2 \times 0.5}{0.34}$

$\mathrm{P}(R|B) = \frac{5}{17},\ 0.294$ *A1 N2*

[3 marks]

Examiners report

[N/A]

3c. In a randomly chosen school week, find the probability that Samantha goes to school by bus on exactly three days. *[2 marks]*

Markscheme

recognizing binomial probability *(R1)*

eg $X \sim \mathrm{B}(n,\ p)$, $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ $(0.34)^3,\ (0.34)^3(1-0.34)^2$

$\mathrm{P}(X=3) = 0.171$ *A1 N2*

[2 marks]

Examiners report

[N/A]

3d. After n school days, the probability that Samantha goes to school by bus at least once is greater than 0.95. Find the smallest value of n. *[5 marks]*

Markscheme

METHOD 1

evidence of using complement (seen anywhere) *(M1)*

eg $1-\mathrm{P}$ (none), $1-0.95$

valid approach *(M1)*

eg $1-\mathrm{P}\text{ (none)} > 0.95,\ \mathrm{P}\text{ (none)} < 0.05,\ 1-\mathrm{P}\text{ (none)} = 0.95$

correct inequality (accept equation) *A1*

eg $1-(0.66)^n > 0.95,\ (0.66)^n = 0.05$

$n > 7.209$ (accept $n = 7.209$) *(A1)*

$n = 8$ *A1 N3*

METHOD 2

valid approach using guess and check/trial and error *(M1)*

eg finding $\mathrm{P}(X \geqslant 1)$ for various values of *n*

seeing the "cross over" values for the probabilities *A1A1*

$n = 7,\ \mathrm{P}(X \geqslant 1) = 0.9454,\ n = 8,\ \mathrm{P}(X \geqslant 1) = 0.939$

recognising $0.9639 > 0.95$ *(R1)*

$n = 8$ *A1 N3*

[5 marks]

Examiners report

[N/A]

4a. A factory makes lamps. The probability that a lamp is defective is 0.05. A random sample of 30 lamps is tested. *[4 marks]*

Find the probability that there is at least one defective lamp in the sample.

Markscheme

evidence of recognizing binomial (seen anywhere) *(M1)*

e.g. $\mathrm{B}(n, p)$, 0.95^{30}

finding $\mathrm{P}(X = 0) = 0.21463876$ *(A1)*

appropriate approach *(M1)*

e.g. complement, summing probabilities

0.785361

probability is 0.785 *A1 N3*

[4 marks]

Examiners report

Although candidates seemed more confident in attempting binomial probabilities than in previous years, some of them failed to recognize the binomial nature of the question in part (a). Many knew that the complement was required, but often used $1 - \mathrm{P}(X = 1)$ or $1 - \mathrm{P}(X \le 1)$ instead of $1 - \mathrm{P}(X = 0)$.

4b. A factory makes lamps. The probability that a lamp is defective is 0.05. A random sample of 30 lamps is tested. *[4 marks]*

Given that there is at least one defective lamp in the sample, find the probability that there are at most two defective lamps.

Markscheme

identifying correct outcomes (seen anywhere) *(A1)*

e.g. $\mathrm{P}(X = 1) + \mathrm{P}(X = 2)$, 1 or 2 defective, $0.3389\ldots + 0.2586\ldots$

recognizing conditional probability (seen anywhere) *R1*

e.g. $\mathrm{P}(A|B)$, $\mathrm{P}(X \le 2|X \ge 1)$, P(at most 2|at least 1)

appropriate approach involving conditional probability *(M1)*

e.g. $\frac{\mathrm{P}(X=1)+\mathrm{P}(X=2)}{\mathrm{P}(X\ge1)}$, $\frac{0.3389\ldots+0.2586\ldots}{0.785\ldots}$, $\frac{1 \text{ or } 2}{0.785}$

0.760847

probability is 0.761 *A1 N2*

[4 marks]

Examiners report

Part (b) was poorly answered. While some candidates recognized that it was a conditional probability, very few were able to correctly apply the formula, identify the outcomes and follow on to achieve the correct result.

Only a few could find the intersection of the events correctly. Several thought the numerator was a product (i.e. $\mathrm{P}(\text{at most } 2) \times \mathrm{P}(\text{at least } 1)$), and then cancelled common factors with the denominator. Others realized that $x = 1$ and $x = 2$ were required but multiplied their probabilities.

This was the most commonly missed out question from Section A.

5a. A bag contains four gold balls and six silver balls. *[8 marks]*

Two balls are drawn at random from the bag, with replacement. Let X be the number of gold balls drawn from the bag.

(i) Find $\mathrm{P}(X = 0)$.

(ii) Find $\mathrm{P}(X = 1)$.

(iii) Hence, find $\mathrm{E}(X)$.

Markscheme

METHOD 1

(i) appropriate approach *(M1)*

eg $\frac{6}{10} \times \frac{6}{10}$, $\frac{6}{10} \times \frac{5}{9}$, $\frac{6}{10} \times \frac{5}{10}$

$P(X = 0) = \frac{9}{25} = 0.36$ *A1* *N2*

(ii) multiplying one pair of gold and silver probabilities *(M1)*

eg $\frac{6}{10} \times \frac{4}{10}$, $\frac{6}{10} \times \frac{4}{9}$, 0.24

adding the product of both pairs of gold and silver probabilities *(M1)*

eg $\frac{6}{10} \times \frac{4}{10} \times 2$, $\frac{6}{10} \times \frac{4}{9} + \frac{4}{10} \times \frac{6}{9}$

$P(X = 1) = \frac{12}{25} = 0.48$ *A1* *N3*

(iii)

$P(X = 2) = 0.16$ (seen anywhere) (A1)

correct substitution into formula for $E(X)$ (A1)

eg $0 \times 0.36 + 1 \times 0.48 + 2 \times 0.16$, $0.48 + 0.32$

$E(X) = \frac{4}{5} = 0.8$ A1 N3

METHOD 2

(i) evidence of recognizing binomial (may be seen in part (ii)) *(M1)*

eg $X \sim B(2, 0.6)$, $\binom{2}{0}(0.4)^2(0.6)^0$

correct probability for use in binomial *(A1)*

eg $p = 0.4$, $X \sim B(2, 0.4)$, ${}^2C_0(0.4)^0(0.6)^2$

$P(X = 0) = \frac{9}{25} = 0.36$ *A1 N3*

(ii) correct set up *(A1)*

eg ${}_2C_1(0.4)^1(0.6)^1$

$P(X = 1) = \frac{12}{25} = 0.48$ *A1* *N2*

(iii)

attempt to substitute into np (M1)

eg 2×0.6

correct substitution into np (A1)

eg 2×0.4

$E(X) = \frac{4}{5} = 0.8$ A1 N3

[8 marks]

Examiners report

Parts (a)(i) and (ii) were generally well done, with candidates either using a tree diagram or a binomial approach. Part (a)(iii) proved difficult, with many either having trouble finding $\mathrm{P}(X = 2)$ or using $\mathrm{E}(X) = np$.

5b. Hence, find $\mathrm{E}(X)$. *[3 marks]*

Markscheme

METHOD 1

$\mathrm{P}(X = 2) = 0.16$ (seen anywhere) ***(A1)***

correct substitution into formula for $\mathrm{E}(X)$ ***(A1)***

eg $0 \times 0.36 + 1 \times 0.48 + 2 \times 0.16$, $0.48 + 0.32$

$\mathrm{E}(X) = \frac{4}{5} = 0.8$ ***A1 N3***

METHOD 2

attempt to substitute into np ***(M1)***

eg 2×0.6

correct substitution into np ***(A1)***

eg 2×0.4

$\mathrm{E}(X) = \frac{4}{5} = 0.8$ ***A1 N3***

[3 marks]

Examiners report

Part (a)(iii) proved difficult, with many either having trouble finding $\mathrm{P}(X = 2)$ or using $\mathrm{E}(X) = np$.

5c. Fourteen balls are drawn from the bag, with replacement. *[2 marks]*

Find the probability that exactly five of the balls are gold.

Markscheme

Let Y be the number of gold balls drawn from the bag.

evidence of recognizing binomial (seen anywhere) ***(M1)***

eg ${}_{14}C_5(0.4)^5(0.6)^9$, $\mathrm{B}(14, 0.4)$

$\mathrm{P}(Y = 5) = 0.207$ ***A1 N2***

[2 marks]

Examiners report

A great majority were confident solving part (b) with the GDC, although some did write the binomial term.

5d. Find the probability that at most five of the balls are gold. *[2 marks]*

Markscheme

recognize need to find $\mathrm{P}(Y \leq 5)$ ***(M1)***

$\mathrm{P}(Y \leq 5) = 0.486$ ***A1 N2***

[2 marks]

Examiners report

Those candidates who did not use the binomial function on the GDC had more difficulty in part (c), although a pleasing number were still able to identify that they were seeking $\mathrm{P}(X \leqslant 5)$.

5e. Given that at most five of the balls are gold, find the probability that exactly five of the balls are gold. Give the answer correct to two decimal places. *[3 marks]*

Markscheme

Let Y be the number of gold balls drawn from the bag.

recognizing conditional probability ***(M1)***

eg $\mathrm{P}(A|B)$, $\mathrm{P}(Y = 5|Y \leq 5)$, $\frac{\mathrm{P}(Y=5)}{\mathrm{P}(Y\leq 5)}$, $\frac{0.207}{0.486}$

$\mathrm{P}(Y = 5|Y \leq 5) = 0.42522518$ ***(A1)***

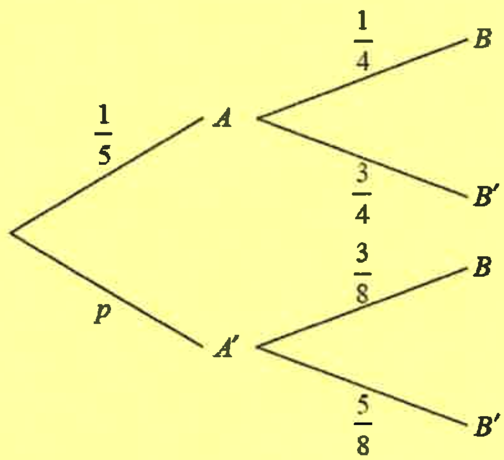
$\mathrm{P}(Y = 5|Y \leq 5) = 0.43$ (to 2 dp) ***A1 N2***

[3 marks]

Examiners report

While most candidate knew to use conditional probability in part (d), fewer were able to do so successfully, and even fewer still correctly rounded their answer to two decimal places. The most common error was to multiply probabilities to find the intersection needed for the conditional probability formula. Overall, candidates seemed better prepared for probability.

6a. The diagram below shows the probabilities for events A and B, with $\mathrm{P}(A') = p$. *[1 mark]*



Write down the value of p.

Markscheme

$p = \frac{4}{5}$ *A1 N1*

[1 mark]

Examiners report

While nearly every candidate answered part (a) correctly, many had trouble with the other parts of this question.

6b. Find $\mathrm{P}(B)$. *[3 marks]*

Markscheme

multiplying along the branches ***(M1)***

e.g. $\frac{1}{5} \times \frac{1}{4}$, $\frac{12}{40}$

adding products of probabilities of two mutually exclusive paths ***(M1)***

e.g. $\frac{1}{5} \times \frac{1}{4} + \frac{4}{5} \times \frac{3}{8}$, $\frac{1}{20} + \frac{12}{40}$

$\mathrm{P}(B) = \frac{14}{40} \left(= \frac{7}{20}\right)$ *A1 N2*

[3 marks]

Examiners report

In part (b), many candidates did not multiply along the branches of the tree diagram to find the required values, and many did not realize that there were two paths for P(*B*). There were also many candidates who understood what the question required, but then did not know how to multiply fractions correctly, and these calculation errors led to an incorrect answer.

6c. Find $\mathrm{P}(A'|B)$. *[3 marks]*

Markscheme

appropriate approach which must include A' (may be seen on diagram) *(M1)*

e.g. $\frac{\mathrm{P}(A'\cap B)}{\mathrm{P}(B)}$ (do not accept $\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}$)

$\mathrm{P}(A'|B) = \frac{\frac{4}{5}\times\frac{3}{8}}{\frac{7}{20}}$ *(A1)*

$\mathrm{P}(A'|B) = \frac{12}{14}\left(=\frac{6}{7}\right)$ *A1 N2*

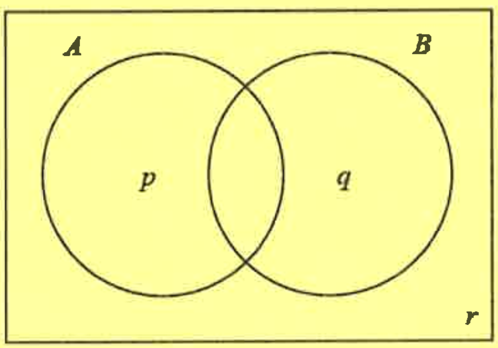
[3 marks]

Examiners report

In part (c), most candidates attempted to use a formula for conditional probability found in the information booklet, but very few substituted the correct values.

7a. Consider the events A and B, where $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.7$ and $\mathrm{P}(A\cap B) = 0.3$. *[3 marks]*

The Venn diagram below shows the events A and B, and the probabilities p, q and r.



Write down the value of

(i) p;

(ii) q;

(iii) r.

Markscheme

(i) $p = 0.2$ *A1 N1*

(ii) $q = 0.4$ *A1 N1*

(iii) $r = 0.1$ *A1 N1*

[3 marks]

Examiners report

As the definitions of p and q were not clear to candidates, both responses of $p = 0.2$, $q = 0.4$ and $p = 0.5$, $q = 0.7$ were accepted for full marks. However, finding r eluded many.

7b. Find the value of $\mathrm{P}(A|B')$. *[2 marks]*

Markscheme

$\mathrm{P}(A|B') = \frac{2}{3}$ ***A2 N2***

Note: Award ***A1*** for an unfinished answer such as $\frac{0.2}{0.3}$.

[2 marks]

Examiners report

Few candidates answered the conditional probability correctly. Many attempted to use the formula in the booklet without considering the complement, and there was little evidence of the Venn diagram being utilized as a helpful aid.

7c. Hence, or otherwise, show that the events A and B are **not** independent. *[1 mark]*

Markscheme

valid reason ***R1***

e.g. $\frac{2}{3} \neq 0.5$, $0.35 \neq 0.3$

thus, A and B are not independent ***AG N0***

[1 mark]

Examiners report

To show the events are not independent, many correctly reasoned that $0.3 \neq 0.35$. A handful recognized that $\mathrm{P}(A|B') \neq \mathrm{P}(A)$ is an alternative approach that uses the answer in part (b). Some candidates do not know the difference between **independent** and **mutually exclusive**.

8a. In a class of 100 boys, 55 boys play football and 75 boys play rugby. Each boy must play at least one sport from football and rugby. *[3 marks]*

(i) Find the number of boys who play both sports.

(ii) Write down the number of boys who play only rugby.

Markscheme

(i) evidence of substituting into $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ *(M1)*

e.g. $75 + 55 - 100$, Venn diagram

30 *A1* *N2*

(ii) 45 *A1* *N1*

[3 marks]

Examiners report

Overall, this question was very well done. There were some problems with the calculation of conditional probability, where a considerable amount of candidates tried to use a formula instead of using its concept and analysing the problem. It is the kind of question where it can be seen if the concept is not clear to candidates.

[4 marks]

8b. One boy is selected at random.

(i) Find the probability that he plays only one sport.

(ii) Given that the boy selected plays only one sport, find the probability that he plays rugby.

Markscheme

(i) **METHOD 1**

evidence of using complement, Venn diagram *(M1)*

e.g. $1 - p$, $100 - 30$

$\frac{70}{100} \left(= \frac{7}{10}\right)$ *A1* *N2*

METHOD 2

attempt to find P(only one sport) , Venn diagram *(M1)*

e.g. $\frac{25}{100} + \frac{45}{100}$

$\frac{70}{100} \left(= \frac{7}{10}\right)$ *A1* *N2*

(ii) $\frac{45}{70} \left(= \frac{9}{14}\right)$ *A2* *N2*

[4 marks]

Examiners report

Overall, this question was very well done. There were some problems with the calculation of conditional probability, where a considerable amount of candidates tried to use a formula instead of using its concept and analysing the problem. It is the kind of question where it can be seen if the concept is not clear to candidates.

[2 marks]

8c. Let A be the event that a boy plays football and B be the event that a boy plays rugby.

Explain why A and B are **not** mutually exclusive.

Markscheme

valid reason in words or symbols *(R1)*

e.g. $\mathrm{P}(A \cap B) = 0$ if mutually exclusive, $\mathrm{P}(A \cap B) \neq 0$ if not mutually exclusive

correct statement in words or symbols *A1* *N2*

e.g. $\mathrm{P}(A \cap B) = 0.3$, $\mathrm{P}(A \cup B) \neq \mathrm{P}(A) + \mathrm{P}(B)$, $\mathrm{P}(A) + \mathrm{P}(B) > 1$, some students play both sports, sets intersect

[2 marks]

Examiners report

In part (c), candidates were generally able to explain in words why events were mutually exclusive, though many gave the wrong values for P(A) and P(B).

8d. Show that A and B are **not** independent. *[3 marks]*

Markscheme

valid reason for independence *(R1)*

e.g. $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$, $\mathrm{P}(B|A) = \mathrm{P}(B)$

correct substitution *A1A1* *N3*

e.g. $\frac{30}{100} \neq \frac{75}{100} \times \frac{55}{100}$, $\frac{30}{55} \neq \frac{75}{100}$

[3 marks]

Examiners report

There was a great amount of confusion between the concepts of independent and mutually exclusive events. In part (d), the explanations often referred to mutually exclusive events.

It was evident that candidates need more practice with questions like (c) and (d).

Some students equated probabilities and number of elements, giving probabilities greater than 1.

9a. A company uses two machines, A and B, to make boxes. Machine A makes 60% of the boxes. *[3 marks]*

80% of the boxes made by machine A pass inspection.

90% of the boxes made by machine B pass inspection.

A box is selected at random.

Find the probability that it passes inspection.

Markscheme

evidence of valid approach involving A and B *(M1)*

e.g. $\mathrm{P}(A \cap \text{pass}) + \mathrm{P}(B \cap \text{pass})$, tree diagram

correct expression *(A1)*

e.g. $\mathrm{P}(\text{pass}) = 0.6 \times 0.8 + 0.4 \times 0.9$

$\mathrm{P}(\text{pass}) = 0.84$ *A1 N2*

[3 marks]

Examiners report

Part (a) was usually well done. Those candidates that did not succeed with this part often did not show a correct tree diagram indicating that they did not really understand the problem or indeed how to start it.

9b. The company would like the probability that a box passes inspection to be 0.87. *[4 marks]*

Find the percentage of boxes that should be made by machine B to achieve this.

Markscheme

evidence of recognizing complement (seen anywhere) *(M1)*

e.g. $\mathrm{P}(B) = x$, $\mathrm{P}(A) = 1 - x$, $1 - \mathrm{P}(B)$, $100 - x$, $x + y = 1$

evidence of valid approach *(M1)*

e.g. $0.8(1 - x) + 0.9x$, $0.8x + 0.9y$

correct expression *A1*

e.g. $0.87 = 0.8(1 - x) + 0.9x$, $0.8 \times 0.3 + 0.9 \times 0.7 = 0.87$, $0.8x + 0.9y = 0.87$

70% from B *A1 N2*

[4 marks]

Examiners report

Many successful attempts to (b) relied on "guess and check" or intuitive solutions while a surprising number of candidates could not manage to systematically set up an appropriate algebraic expression involving a complement.