Calculus Review p. 327

NON-CALCULATOR

1) a) $\int(4x^3-8x+6)dx = 4\left(\frac{1}{4}\right)x^4 - 8\left(\frac{1}{2}\right)x^2+6x+C = x^4-4x^2+6x+C$

b) $\int(\sqrt[3]{x})^4dx = \int x^{4/3}dx = \frac{3}{7}x^{\frac{4}{3}+\frac{3}{3}=\frac{7}{3}}+C = \frac{3}{7}x^{\frac{7}{3}}+C$

c) $\int \frac{3}{x^4}dx = 3\int x^{-4}dx = 3\left[-\frac{1}{3}x^{-3}\right]+C = -\frac{1}{x^3}+C$

d) $\int \frac{5x^4-3x}{6x^2}dx = \int\left(\frac{5}{6}x^2 - \overset{\frac{1}{2}x^{-1}}{\frac{1}{2x}}\right)dx = \frac{5}{6}\left(\frac{1}{3}\right)x^3 + \frac{1}{2}\ln(2x)+C$

$= \frac{5}{18}x^3 + \frac{1}{2}\ln(2x)+C$

e) $\int e^{4x}dx = \frac{1}{4}e^{4x}+C$

f) $\int x^2(x^3+1)^4\,dx = \frac{1}{3}\int u^4\,du = \frac{1}{3}\left(\frac{1}{5}\right)u^5+C = \frac{1}{15}(x^3+1)^5+C$

let $u = x^3+1$

$du = 3x^2dx$

$\frac{1}{3}du = x^2dx$

g) $\int \frac{1}{2x+3}dx = \frac{1}{2}\int\frac{1}{u}du = \frac{1}{2}\ln(u)+C = \frac{1}{2}\ln(2x+3)+C$

let $u = 2x+3$

$du = 2dx$

h) $\int \frac{\ln x}{x}dx = \int \ln x\left(\frac{1}{x}\right)dx = \int u\,du = \frac{1}{2}u^2+C = \frac{1}{2}\cdot\left(\frac{1}{x}\right)^2+C = \frac{1}{2x^2}+C$

$u = \frac{1}{x}$

$du = \ln x\,dx$

i) $\int(3x^2+1)(6x)dx = \int u\,du = \frac{1}{2}u^2+C = \frac{1}{2}(3x^2+1)+C$ or $\frac{3}{2}x^2+\frac{3}{2}+C$

$u = 3x^2+1$

$du = 6x\,dx$

j) $\int\frac{2e^x}{e^x+3}dx = 2\int\frac{1}{u}du = 2\ln(u)+C$

$= 2\ln(e^x+3)+C$

$u = e^x+3$

$du = e^x dx$

1) k) $\int 3\sqrt{2x-5}\,dx = 3\int (2x-5)^{1/2}dx = 3\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)(2x-5)^{3/2} + C = (2x-5)^{3/2} + C$

l) $\int 2x e^{2x^2} dx \quad = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \boxed{\frac{1}{2}e^{2x^2} + C}$

$\cancel{u = e^{2x^2}}$

$\cancel{du = e^{2x^2}(4x)dx}$

Nope – try again

$u = 2x^2$

$du = 4x\,dx$

$\boxed{\frac{1}{2}du = 2x\,dx}$

2)
a) $\int_0^2 (3x^2 - 6)dx = \left[x^3 - 6x\right]_0^2 = (2)^3 - 6(2) - ((0)^3 - 6(0)) = 8 - 12 \boxed{= -4}$

b) $\int_4^{16} 4t^{-1/2}dt = 4\int_4^{16} t^{-1/2}dt = 4\left[\frac{2}{1}t^{1/2}\right]_4^{16} = 4\left[2(\sqrt{16}) - 2\sqrt{4}\right]$

$= 4(2\cdot 4 - 2\cdot 2)$

$\boxed{= 4(8-4) = 4(4) = 16}$

c) $\int_1^{e^2} \frac{4}{x}dx = 4\int_1^{e^2} \frac{1}{x}dx = \left[4\ln(x)\right]_1^{e^2} = 4\left(\ln(e^2) - \ln(1)\right)$

$\boxed{= 4(2 - 0) = 8}$

d) $\int_0^1 6x e^{3x^2+3}dx \quad = \int_3^6 e^u du = \left[e^u\right]_3^6 = \left[e^{3x^2+3}\right]_3^6 = e^{3(6)^2+3} - e^{3(3)^2+3}$

$u = 3x^2 + 3$ when $x = 1$, $u = 3(1)^2 + 3 = 6$

$du = 6x\,dx$ when $x = 0$, $u = 3(0)^2 + 3 = 3$

$\boxed{= e^{111} - e^{30}}$

e) $\int_{-1}^1 (3x-1)^3 dx = \left[\frac{1}{3}\left(\frac{1}{4}\right)(3x-1)^4\right]_{-1}^1 = \frac{1}{12}\left[(3(1)-1)^4 - (3(-1)-1)^4\right]$

$= \frac{1}{12}(16 - 256) = \boxed{-20.75}$

or → $u = 3x - 1$, $du = 3x$ $\quad \frac{1}{3}\int_{-4}^2 u^3 du = \left[\frac{1}{3}\left(\frac{1}{4}\right)u^4\right]_{-4}^2 = \frac{1}{12}\left[(2)^4 - (-4)^4\right] = \boxed{-20.75}$

$x = -1$, $u = -4$

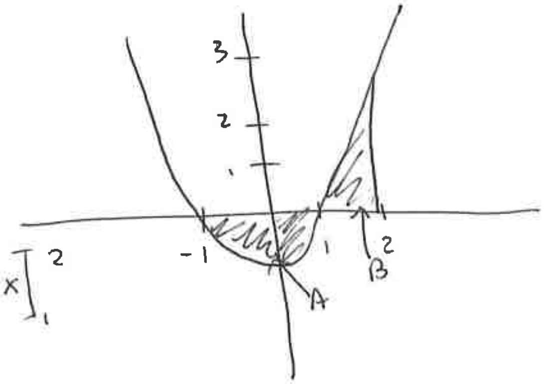
$x = 1$, $u = 2$

f) $\int_0^2 \frac{1}{2x+1}dx = \left[\ln(2x+1)\right]_0^2 = \ln(2(2)+1) - \ln(2(0)+1)$

$= \ln(5) - \ln(1)$

$\boxed{= \ln(5)}$

3) $f(x) = x^2 - 1$



a) $A_B = \int_1^2 (x^2-1)\,dx$

b) $\int_1^2 (x^2-1)\,dx = \left[\frac{1}{3}x^3 - x\right]_1^2$

$= \frac{4}{3}$

c) $\int_{-1}^2 A_A + A_B$

$\int_{-1}^2 (x^2-1)\,dx =$

d) $\int_{-1}^2 \pi (x^2-1)^2$

4) $y = f(x)$ $(2,6)$ $f'(x) = 3x-2$

$\int (3x-2)\,dx = \frac{3}{2}x^2 - 2x + C$

then $6 = \frac{3}{2}(2)^2 - 2(2) + C$

$6 = 6 - 4 + C$

$6 = 2 + C$

$4 = C$

$\therefore f(x) = \frac{3}{2}x^2 - 2x + 4$

5) $\int_1^5 f(x)\,dx = 20$

a) $\int_1^5 \frac{1}{4} f(x)\,dx = \frac{1}{4}\int_1^5 f(x)\,dx = \frac{1}{4}(20) = 5$

b) $\int_1^5 [f(x) + 2]\,dx$

6) $v(t) = 4e^{2t} + 2$ $(0,8)$

$\int (4e^{2t} + 2)\,dt = 4\int e^{2t}\,dt + 2\int 1\,dt$ $\boxed{s(t) = 2e^{2t} + 2t + 6}$

$= 4\left(\frac{1}{2}\right)e^{2t} + 2t + C$

$8 = 2e^{2(0)} + 2(0) + C$

$8 = 2e^0 + C$

$8 = 2(1) + C$

$8 = 2 + C$

$\boxed{6 = C}$

7) Given $\int_1^K \frac{1}{2x-1}\,dx = \ln 5$, find K

$$\int_1^K \frac{1}{2x-1}\,dx = \left[\tfrac{1}{2}\ln(2x-1)\right]_1^K = \tfrac{1}{2}\ln(2K-1) - \tfrac{1}{2}\ln(2(1)-1)$$
$$= \tfrac{1}{2}\ln(2K-1) - \underbrace{\tfrac{1}{2}\ln(1)}_{0}$$

$$\ln 5 = \tfrac{1}{2}\ln(2K-1)$$
$$\cancel{\ln} 5 = \cancel{\ln}\sqrt{2K-1}$$
$$5^2 = 2K-1$$
$$25 = 2K-1$$
$$26 = 2K$$
$$\boxed{K=13}$$

Calculator Review

1) $\int_a^b \pi(4-x^2)^2\,dx$

$4-x^2=0$
$4=x^2$
$x=\pm 2$

$\pi\int_{-2}^{2}(4-x^2)^2\,dx = 107.2$

2) $v(t)=2t^2-11t+12 \longrightarrow 2t^2-11t+12=0$
$t=1.5, 4$

a) $a(t)=4t-11$

b) $a<t<b$ moving left

$4t-11=0$
$4t=11$

$a=1.5,\ b=4$

c) $\int_2^5 |2t^2-11t+12|\,dt = 7.83$ m

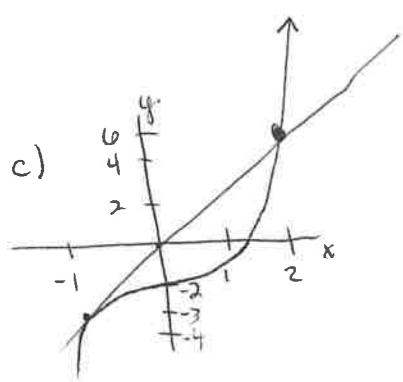
3) $f(x)=x^3-2 \Rightarrow f(-1)=-3$

a) $f'(x)=3x^2 \Longrightarrow m=f'(-1)=3$

$y+3=3(x+1)$
or
$y=3x$

b) $x^3-2=3x$
$x=2$
$f(2)=2^3-2=6$
point is $(2,6)$

c)



d) $\int_{-1}^{2}[3x-(x^3-2)]\,dx = 6.75$