Series and Sequences Review

Answer Key

1a. *[2 marks]*

In an arithmetic sequence, the first term is 2 and the second term is 5.

Find the common difference.

$u_1 = 2, \quad u_2 = 5$

$d = 3$

1b. *[2 marks]*

Find the eighth term.

$u_8 = 2 + 3(7) = 23$

1c. *[2 marks]*

Find the sum of the first eight terms of the sequence.

$S_8 = \frac{8}{2}(2 + 23) = 100$

2a. *[3 marks]*

In a geometric series, $u_1 = \frac{1}{81}$ and $u_4 = \frac{1}{3}$.

Find the value of r.

$u_4 = u_1 r^3$

$\frac{1}{3} = \frac{1}{81} r^3$

$\frac{81}{3} = r^3$

$27 = r^3$

$3 = r$

2b. *[4 marks]*

Find the smallest value of n for which $S_n > 40$.

$S_n = \frac{u_1(1-r^n)}{1-r^n} = \frac{\frac{1}{81}(1-3^n)}{1-3^n} = 8$

3. *[6 marks]*

$n=3$

The sum of the first three terms of a geometric sequence is 62.755, and the sum of the infinite sequence is 440. Find the common ratio.

$$S_\infty = \frac{u_1}{1-r} = 440$$

u_1

$$u_1 = 440(1-r)$$

$$\frac{u_1(1-r^3)}{(1-r)} = 62.755$$

$$\frac{440\cancel{(1-r)}(1-r^3)}{\cancel{(1-r)}} = 62.755$$

$$440(1-r^3) = 62.755$$

$$1-r^3 = \frac{62.755}{440}$$

$$-r^3 = \frac{62.755}{440} - 1$$

$$r = \sqrt[3]{\frac{62.755}{440} - 1} = 0.95$$

4a. *[2 marks]*

An arithmetic sequence, $u_1, u_2, u_3 \ldots$, has $d = 11$ and $u_{27} = 263$.

Find u_1.

$$u_n = u_1 + d(n-1)$$

$$263 = u_1 + 11(27-1)$$

$$263 = u_1 + 286$$

$$u_1 = -23$$

4b. *[4 marks]*

(i) Given that $u_n = 516$, find the value of n.

$$u_n = u_1 + d(n-1)$$

$$516 = -23 + 11(n-1)$$

$$516 = -23 + 11n - 11$$

$$516 = -34 + 11n$$

$$550 = 11n$$

$$50 = n$$

(ii) For this value of n, find S_n.

$$S_{50} = \frac{50}{2}(\underbrace{-23}_{u_1} + \underbrace{516}_{u_n}) = 12325$$

or 12300 for 3sf.

5a. *[5 marks]*

The first two terms of a geometric sequence u_n are $u_1 = 4$ and $u_2 = 4.2$.

(i) Find the common ratio.

$$r = \frac{u_2}{u_1} = \frac{4.2}{4} = 1.05$$

(ii) Hence or otherwise, find u_5. $u_n = u_1 r^{n-1}$

$$u_5 = 4(1.05)^4 \approx 4.86$$

6a. *[1 mark]*

Expand $\sum_{r=4}^{7} 2^r$ as the sum of four terms.

$$\sum_{r=4}^{7} 2^r = 2^4 + 2^5 + 2^6 + 2^7$$

6b. *[6 marks]*

(i) Find the value of $\sum_{r=4}^{30} 2^r$

$$S_n = \frac{u_1(1-r^n)}{(1-r)}, \quad r \neq 1$$

$$u_1 = 2^4, \quad r = 2, \quad \sum_{r=4}^{30} = \sum_{r=1}^{30} - \sum_{r=1}^{3} \Rightarrow n = 27$$

$$\sum_{r=1}^{30} 2^r = \frac{2(1-2^{30})}{1-2} = 2147483646$$

$$\sum_{r=1}^{3} 2^r = \frac{2(1-2^3)}{1-2} = 14$$

$$\sum_{r=4}^{30} 2^r = 2147483646 - 14 = \boxed{2147483632}$$

(ii) Explain why $\sum_{r=4}^{\infty} 2^r$ cannot be evaluated.

Since $|r| > 1$, the series will continue to grow (diverge)

7a. *[1 mark]*

Consider the expansion of $(x+3)^{10}$.

Write down the number of terms in this expansion.

$n+1 = 11$ term

7b. *[4 marks]*

Find the term containing x^3. $x^0 \quad x^1 \quad x^2 \quad x^3$

$\binom{10}{3}x^3(3)^7$

$\binom{10}{3}x^3(3)^7 = 120x^3(2187) = 262440x^3$

8. In the expansion of $(3x-2)^{12}$, the term in x^5 can be expressed as $\binom{12}{r} \times (3x)^p \times (-2)^q$.

$\binom{12}{5}(3x)^5(-2)^7$

8a. *[3 marks]*

Write down the value of p, of q and of r.

$p=5$

$q=7$

$r=5$

8c. *[2 marks]*

Find the coefficient of the term in x^5.

$\binom{12}{5}(3x)^5(-2)^7 = (792)(243)x^5(-128)$

$= \boxed{-24634368}x^5$

coefficient