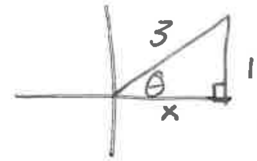
Practice for Unit Circle Part 1

1. *[6 marks]*

Given that $\sin x = \frac{1}{3}$, where $0 < x < \frac{\pi}{2}$, find the value of $\cos 4x$.



$x^2 + 1 = 9$

$x^2 = 8$

$x = 2\sqrt{2}$

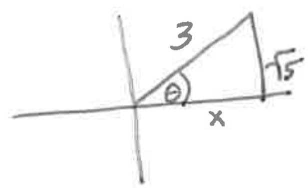
$$\cos(4x) = \cos^2(2x) - \sin^2(2x)$$
$$= (\cos(2x))^2 - (\sin(2x))^2$$
$$= (1 - 2\sin^2(x))^2 - (2\sin x \cos x)^2$$
$$= 1 - 4\sin^2(x) + 4\sin^4 x - 4\sin^2 x \cos^2 x$$
$$= 1 - 4\left(\tfrac{1}{3}\right)^2 + 4\left(\tfrac{1}{3}\right)^4 - 4\left(\tfrac{1}{3}\right)^2\left(\tfrac{\sqrt{8}}{3}\right)^2$$
$$= 1 - \frac{4}{9} + \frac{4}{81} - \left(\frac{4}{9}\right)\left(\frac{8}{9}\right)$$
$$= 1 - \frac{36}{81} + \frac{4}{81} - \frac{32}{81} = \frac{17}{81}$$

$\boxed{OR}$

$$\cos(2x) = 2\cos^2 x - 1$$
$$= 2\left(\tfrac{\sqrt{8}}{3}\right)^2 - 1 = \frac{7}{9}$$
$$\cos(4x) = 2\left(\tfrac{7}{9}\right)^2 - 1$$
$$= \frac{17}{81}$$

2a. *[3 marks]*

Let $\sin\theta = \frac{\sqrt{5}}{3}$, where θ is acute.



$x^2 + (\sqrt{5})^2 = 3^2$

$x^2 + 5 = 9$

$x^2 = 4$

$x = 4$

Find $\cos\theta$.

$$\cos\theta = \frac{4}{3}$$

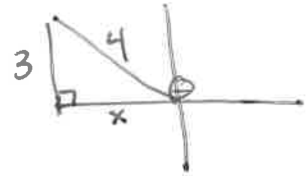
2b. *[2 marks]*

Find $\cos 2\theta$. $= \cos^2\theta - \sin^2\theta$

$$= \left(\tfrac{4}{3}\right)^2 - \left(\tfrac{\sqrt{5}}{3}\right)^2$$
$$= \frac{16}{9} - \frac{5}{9} = \frac{11}{9}$$

3a. *[4 marks]*

Given that $\sin x = \frac{3}{4}$, where x is an obtuse angle,



find the value of $\cos x$;

$\cos x = \frac{\sqrt{7}}{4}$

$x^2 + 3^2 = 4^2$

$x^2 = 7$

$x = \sqrt{7}$

3b. *[3 marks]*

find the value of $\cos 2x$.

$\cos(2x) = \cos^2 x - \sin^2 x$

$= \left(\frac{\sqrt{7}}{4}\right)^2 - \left(\frac{3}{4}\right)^2 = \frac{7}{16} - \frac{9}{16} = -\frac{2}{16} = -\frac{1}{8}$

4. *[7 marks]*

Solve $\cos 2x - 3\cos x - 3 - \cos^2 x = \sin^2 x$, for $0 \leq x \leq 2\pi$.

$\cos 2x - 3\cos x - 3 - \cos^2 x - \sin^2 x = 0$

$2\cos^2 x - 1 - 3\cos x - 3 - (\cos^2 x + \sin^2 x) = 0$

$2\cos^2 x - 1 - 3\cos x - 3 - 1 = 0$

$2\cos^2 x - 3\cos x - 5 = 0$

$\frac{-3 \pm \sqrt{(-3)^2 - 4(2)(-5)}}{2(2)}$

$\frac{-3 \pm \sqrt{9 - 40}}{4}$

$\cos^2 x - \sin^2 x - 3\cos x - 3 - \cos^2 x - \sin^2 x = 0$

$-2\sin^2 x - 3\cos x - 3 = 0$

$-2(1 - \cos^2 x) - 3\cos x - 3 = 0$

$-2 + 2\cos^2 x - 3\cos x - 3 = 0$

$2\cos^2 x - 3\cos x - 5 = 0$

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QUIZ: Verifying Trigonometric Identities

Name ______________________________ Period __________ Date __________

Directions: **Verify each identity using trigonometric identities.**

1. $\dfrac{\csc x}{\sec x} = \cot x$

$$LHS = \frac{\frac{1}{\sin x}}{\frac{1}{\cos x}} = \frac{1}{\sin x} \cdot \frac{\cos x}{1} = \frac{\cos x}{\sin x} = \cot x$$
$$= RHS \checkmark$$

2. $\sec x \cot x = \csc x$

$$LHS = \left(\frac{1}{\cos x}\right)\left(\frac{\cos x}{\sin x}\right)$$
$$= \frac{1}{\sin x} = \csc x = RHS \checkmark$$

3. $(1 - \cos^2 x)\csc^2 x = 1$

$$LHS = (\sin^2 x)\left(\frac{1}{\sin^2 x}\right)$$
$$= 1 = RHS \checkmark$$

4. $(1 + \tan^2 x)\sin^2 x = \tan^2 x$

$$LHS = \sec^2 x \sin^2 x$$
$$= \frac{1}{\cos^2 x} \cdot \sin^2 x$$
$$= \frac{\sin^2 x}{\cos^2 x}$$
$$= \tan^2 x = RHS \checkmark$$

5. $\dfrac{\sin x \cos x}{(1+\cos x)(1-\cos x)} = \cot x$

6. $\cot x - \cos x = \cot x(1-\sin x)$

7. $\dfrac{\sec x}{\sin x} - \dfrac{\sin x}{\cos x} = \cot x$

8. $\tan^2 x - \tan^2 x \sin^2 x = \sin^2 x$